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Analysis of Water supply Networks

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الخلاصة:

في هذا الورقة ، تم دراسة وتحليل شبكة توزيع المياه داخل الأنابيب، و التي تتكون من أربعة حلقات (loops) و 12 أنبوب ، بطاقة 100 m، باستخدام ثـلاث طرق هاردي كـروس ،النظريـة الخطية و كـذلك طريـقة المـيل. منهجية حـل هـــذه الطرق في ظل ظروف الحالة المستقرة (steady-state) . هذه المنهجية مقترحة على أساس تقييم وتحليل هذه الطرق الثلاث، ومقارنـاتهم مع بعضهم تحت تلك الظروف لتشكيل نتائج محـاكـاة. كـذلك تـم استخدام برنـامج الجدولة لتطوير خـوارزمية هـاردي كروس، بـالإضافة إلـى إعـادة صيـاغة معـادلات الحـلقة و الـعقد (loop &node equations) لقانون كيرتشهوف الأول و الثاني باستخدام المصفوفات لحل كلا الطريقتين النظرية الخطية و الميل، باستعمال برنـامج الجدولة.(Microsoft Excel) . على أية حال، طريقة هاردي كروس كانت الطريقة الأساسية المستخدمة بشكل واسع و لعدة سنوات لتحليل شبكات تغذية المياه، حيث تم الحصول على النتائج والحلول بواسطة برنامج الجدولة المتوفرة عموما، التي تزود المعلمين و الطلاب ببديل رخيص مناسب للأغراض الأكاديمية و العملية بعيداً عن البرامج المعقدة. النتائج كانت مرضية و مقنعة لنموذج شبكة توزيع المياه المقترح لكلا الطريقتين النظرية الخطية و الميل بالنسبة لأداء التقارب لمعدَلات التصريف وسمت الطاقة (pressure head elevations) ، على عكس طريقة هاردي كروس غير المتكافئة نسبيًا. هذه الطرق ، بما في ذلك طريقة النظرية الخطية وطريقة الميل ، تختلف هذه الطرق عن طريقة هاردي كروس لأنها تأخذ في الحسبان جميع حلقات الشبكة في وقت واحد وبالتالي تتقارب بشكل عام في عدد أقل من التكرارات. علاوة على ذلك ، تستخدم معظم برامج النمذجة الهيدروليكية الحالية الآن طرقًا بديلة أكثر كفاءة مثل طريقة نيوتن رافسون، وطريقة النظرية الخطية ، وطريقة الميل.

Abstract

In this paper, a supply water network was studied and analyzed using three methods, namely; Hardy Cross method, Linear Theory and Gradient method under the same conditions. Results were compared and interpreted. Four loops, 12 pipes with 100 m energy head were considered. Methodology of the solution these methods were under steady-state conditions. This Methodology is proposed on the basic of an evaluation and analysis three methods and comparing them to each other under the same conditions of forming simulation results. A spreadsheet solution was also developed for the Hardy Cross algorithm, also reformulation of the loop and the node equations of Kirchhoff's first and second law using the matrixes for both of the two methods, the linear theory and gradient method using Microsoft Excel. However, the Hardy Cross is considered the famous method the primary method widely used in steady-state analyses of water supply networks for many years, the results and solutions were generally obtained by the aid of computers(MS Excel) which provide teachers and students with affordable alternatives suitable for educational and practical purposes against complicated programs. The results were satisfactory and convincing for the proposed water distribution network model for the Linear Theory and the Gradient Method relative to the Convergence Performances of the discharge rates and pressure head elevations , unlike Hardy Cross method is relatively uneven. These methods, including the Linear Theory method, and Gradient method, all these methods vary from the Hardy Cross method because they take into account all loops simultaneously and therefore generally converge in fewer iterations. Further, most current hydraulic modeling software now uses alternative methods which are more efficient as the Newton-Raphson method, Linear Theory method, and Gradient method.

Keywords: Teaching methods; Hydraulic networks; Water distribution; supply networks; Computer software; Spreadsheets; Correction; Iteration.

1. Introduction

The hydraulic model of water distribution networks is one of the practical facts that provide the reader with a comprehensive and relatively practical study for students of civil and environmental engineering, irrigation and agricultural engineering, and mechanical engineering. The fluid flow through pipelines has applications, that include transporting water over long distances to supply cities and rural towns and so forth. Studying methods of analyzing water supply networks and comparing them is noteworthy in terms of the quality of the method and its ability to respond to different designs. Therefore, predicting the flow rates and pressures through the network, especially that contains a number of branches and interlocking rings, is one of the problems related to hydraulic design, which is known as the analysis of pipe networks. The objective of this analysis is to monitor the flow rates, and to determine the degrees of pressure drop within the network.

Infrastructure of water supply varies in its complexity from a simple, rural town gravity system to a computerized, remote-controlled, multisource system of a large city or country for example man-made river system in Libya ; however, the aim and objective of all the water systems are to supply safe water for the cheapest cost. These systems are designed based on least-cost and depended reliability considerations.

In order to predict the system reaction against any conditional change, a series of simulations must be done. Several approaches have been suggested for the solution of distribution systems including Linear Theory, Hardy-Cross and Gradient Algorithm [3]. All these methodologies require a detailed examination of water distribution system. Besides, these methodologies require great amounts of calculations for the solution of the system, which is time consuming if hand calculations are employed. Therefore, computer programs are used in hydraulic simulation of looped networks.

Majid Niazkar And Seied Hosein Afzali [4] implemented Q-based methods in MATLAB and Excel spreadsheet. They pointed out that MATLAB and Excel spreadsheet provide suitable facilities for both academic and practical purposes, the comprehensive application of these programs in water distribution networks analysis has not been addressed. Moreover, they reasoned to focus more on the educational aspects of computer application. In addition to, as basics of the implementation are sufficiently covered, the provided codes can be improved analyze more complicated pipe networks.

David H. Huddleston, P.E., M.Asce1, And Vladimir J. Alarcon [5] analyzed Hardy-Cross method using Excel. They concluded that the application of commonly available spreadsheet software (Microsoft Excel) to more concisely and effectively solve typical undergraduate network distribution problems using linear theory. Application development is much more efficient and straightforward than the corresponding Hardy Cross implementation enabling students to concentrate upon the engineering system and relevant design issues.

Selami Demir et al. [6] evaluated in their study of the modified Hardy-Cross method that it has proved to be an accurate tool for time-dependent simulation of water distribution networks. The newly developed algorithm for method is able to perform both steady-state and time-dependent simulations of water distribution systems. Also a computer program was developed. Both of two solution algorithms were implemented in MS Excel Macros enabling the user to select between

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steady-state and time-dependent simulations. Besides, the validity of the new method was tested for an example system of 21 pipes against EPANET. The new program and EPANET showed no difference at all.

Dejan Brkić and Pavel Praks [7] presented the node-loop method which is the powerful numerical procedure for calculation of flows or diameters as inverse problems in looped fluid distribution networks. They concluded main advantages is that flow in each pipe can be calculated directly, which is not possible after Hardy Cross and improved Hardy Cross methods. Also recommended the similar numbers of iterations are necessary to achieve demanded accuracy in calculation as in the modified Hardy Cross method.

2. Analysis of water distribution network

Figure (1) represents the layout of water distribution network and the model for this study. This Model contains 9 nodes four loops and 12 pipes which exceeds the usual expectations of a class assignment via manual calculations. All piping materials are assumed to be concrete. Specified nodes demands and the assumed flow direction are shown on the figure. The Hazen-Williams friction model is applied throughout the network.

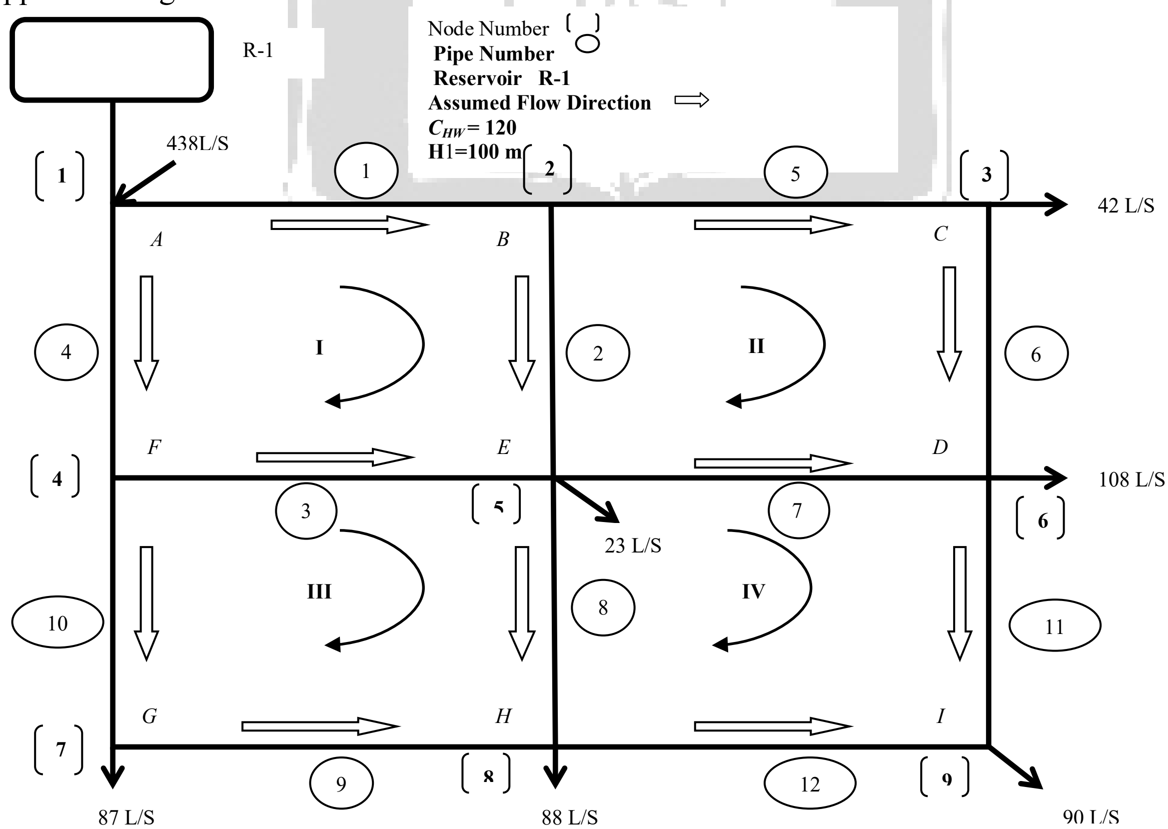

Figure (1) Schematic drawing for the water distribution network

The piping network is analyzed by developing a system of equations that represent the conservation of mass enforced at each of the 9 nodes, conservation of energy for each of the four network loops. This yields a system consists of 12 and 20 nonlinear, algebraic equations to solve simultaneously for the 12 unknown volumetric flow rates. Reservoir is R-1 and energy head at node no.1 pipeline data are listed in Table 1.

Generally, the governing equations are mainly casted in two different ways: (1) Q-based (Hardy Cross method, Linear Theory, and Newton-Raphson method) and (2) h-based (Newton-Rapshon method, finite element, and Gradient method).This different methods can be valid for solving Water Distribution Network for different casting.

Table 1. Pipe line Data for Water Distribution Network.

Pipe number [ ]	Diameter (m)	Length (m)	Hazen-Williams Calculated resistance factor at C_{HW} =120 K_i	Pipe number [ ]	Diameter (m)	Length (m)	Hazen-Williams Calculated resistance factor at C_{HW} =120 K_i
1	0.508	0915	37.377	7	0.305	0915	448.354
2	0.406	1220	148.449	8	0.305	1220	597.805
3	0.406	0915	111.337	9	0.406	0915	111.337
4	0.61	1220	20.443	10	0.406	1220	148.449
5	0.508	0915	37.377	11	0.305	1220	597.805
6	0.406	1220	148.449	12	0.305	0915	448.354

2.1. The Hardy Cross Method and its Successors in Water Distribution Modeling: Q-Based Methods

In this paper, the governing equations are casted for the three Q-based methods. Since the governing equations under the steady-state condition comprise a nonlinear algebraic system of equations, an iteration based scheme using initial guesses for state variables in each method to solve water supply network. The description and formulations of these methods are presented in the following:

2.1.1. Hardy cross method

Loop equations .The loop equations express conservation of mass and energy in terms of the pipe flows. Mass must be conserved at a node, as for all N_j junction nodes in a network, it can be written as

$$\sum Q_{in} = Q_{out} \quad for\ all\ N_j\ \ nodes \tag{1}$$

where Q_{in} and Q_{out} denote nodal demands and pipe flows into and out of the junction node. conservation of energy requires that the sum of energy head loss h_L in each of the I_P pipes and energy gain H_{pump} across each of the J_P pumps in the loop must balance the net change in energy head ΔE_{FGN} as

$$\sum_{i=1,I_p} h_{l,i} - \sum_{ip=1,I_p} H_{Pump,ip} - \Delta E_{FGN} = 0 \tag{2}$$

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Equation (2) simplifies this case by dropping the pump terms and setting ΔE to zero. Compute frictional and, if any, minor head losses through all of the pipes.

$$h_{L,i} = K_1 Q_i^{n1} + K_2 Q_i^{n2} \qquad (3)$$

The loop flow rate correction parameters are calculated using expression:

$$\Delta Q_j = \frac{-\sum_{i=1,Ip} K_i Q_i^n}{n \sum_{i=1,Ip} \left| K_i Q_{i,k}^{n-1} \right|} = \frac{-\sum_{i=1,Ip} h_{l,i}}{n \sum_{i=1,Ip} \left| h_{l,i} / Q_i \right|} \qquad (4)$$

Once the correction has been computed, the estimates for the next iteration are computed by

$$Q_{i,k+1} = Q_i + \Delta Q_j \qquad (5)$$

Figure 2 shows the flowcharts for the application of iterative Steady-State Hardy-Cross methodology analysis.

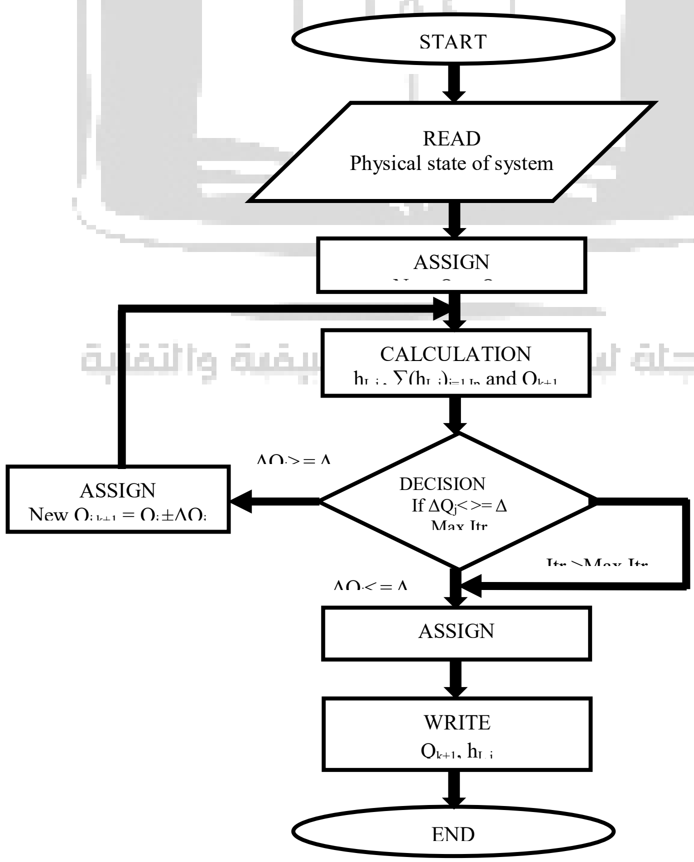

Figure 2 steady-state analysis

TABLE 2 – Calculation of Pipe Resistance Factors and Power Coefficients

Function Method	K_1	n_1	K_2	n_2
Darcy-weisbach	$\frac{8}{g\pi^2 D_i^4}\left[\frac{f_i L_i}{D_i}+\sum_{j=1}^{n} R_{i,j}\right]$	2	0	0
Hazen-Williams	$\frac{(10.6331)L_i}{(C_{HW})_i^{1.85} D_i^{4.87}}$	1.85	$\frac{0.083}{D_i^4}\sum_{j=1}^{n} R_{i,j}$	2
Manning*	$\frac{10.29 n_i^2 L_i}{D_i^{5.33}}+\frac{0.083}{D_i^4}\sum_{j=1}^{n} R_{i,j}$	2	0	0

* Although Manning's equation is mostly used for surface flows, it is sometimes allowed for the use for pipe flows.

2.1.2. Linear theory method

Linear theory solves the loop equations or Q equations (Eqs.1 to 2). N_p equations ($N_j + N_l + N_f - 1$) can be written in terms of the N_p unknown pipe flows. Since these equations are nonlinear in terms of Q, an iterative procedure is applied to solve for the flows. Linear theory, as described in Wood and Charles (1972), linearizes the energy equations about $Q_{i,k+1}$, where the subscript $k+1$ denotes the current iteration number using the previous iterations $Q_{1,k}$ as known values. Considering only pipes in this derivation[8], these equations are

$$\sum_{i=1,I_p}^{m} Q_{i,k+1} = Q_{ext} \qquad for\ all\ Nj\ nodes \tag{6}$$

$$\sum_{i=1,I_p}^{m} K_i\, Q_{i,k}^{n-1}\, Q_{i,k+1} = 0 \qquad for\ all\ N_l\ closed\ loops \tag{7}$$

$$\sum_{i=1,I_p}^{m} K_i\, Q_{i,k}^{n-1}\, Q_{i,k+1} = \Delta E_{FGN} \qquad for\ all\ Nf-1\ independent\ pseudo-loops \tag{8}$$

Where K is Pipe resistance constant, i is Pipe number ,k is Iteration , Q is Discharge ,and n is Coefficient expressing the relationship between flow and head loss. For all nodes and in pipes the network from figure 1, the flow equations at nodes 2, 3, 4, 5,6,7,8 and 9 are written using Kirchoff's first law and the head equations around loops 1, 2,3 and loop 4 are written using Kirchoff's Second Law in the form of equation 8. Can be noted as following

Node equations

Node2: $Q_1 - Q_5 - Q_2 = 0$

Node3: $Q_5 - Q_6 = 0.042$

Loop equations

Loop1: $K_1 Q_1^{n-1} + K_2 Q_2^{n-1} - K_3 Q_3^{n-1} - K_4 Q_4^{n-1}$

Loop2: $-K_2 Q_2^{n-1} + K_5 Q_5^{n-1} + K_6 Q_6^{n-1} - K_7 Q_7^{n-1}$

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Node4: $Q_4 - Q_3 - Q_{10} = 0$

Node5: $Q_2 + Q_3 - Q_7 - Q_8 = 0.023$

Node6: $Q_6 + Q_7 - Q_{11} = 0.108$

Node7: $Q_9 + Q_{10} = 0.087$

Node8: $Q_8 + Q_9 - Q_{12} = 0.088$

Node9: $Q_{11} + Q_{12} = 0.09$

Loop3: $K_3 Q_3^{n-1} + K_8 Q_8^{n-1} - K_9 Q_9^{n-1} - K_{10} Q_{10}^{n-1}$

Loop4: $K_7 Q_7^{n-1} - K_8 Q_8^{n-1} + K_{11} Q_{11}^{n-1} -$ $nK_{12} Q_{12}^{n-1}$

This system above of equations used in linear theory is illustrated in equation (9) in the matrix format for the sample network

$$\begin{bmatrix}
1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
K_1Q_1^{n-1} & K_2Q_2^{n-1} & -K_3Q_3^{n-1} & -K_4Q_4^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -K_2Q_2^{n-1} & 0 & 0 & K_5Q_5^{n-1} & K_6Q_6^{n-1} & -K_7Q_7^{n-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_3Q_3^{n-1} & 0 & 0 & 0 & 0 & K_8Q_8^{n-1} & -K_9Q_9^{n-1} & -K_{10}Q_{10}^{n-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K_7Q_7^{n-1} & K_8Q_8^{n-1} & 0 & 0 & K_{11}Q_{11}^{n-1} & -K_{12}Q_{12}^{n-1}
\end{bmatrix}
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0.042 \\ 0.108 \\ 0 \\ 0.023 \\ 0.087 \\ 0.088 \\ 0.09 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

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2.1.3. Gradient method

The Gradient method uses the same principles as the Newton-Raphson method, but simultaneously solves for both flow and head values. It is considered a variant of the Newton-Raphson method and is a *gradient / node-loop method*. Unlike the Newton-Raphson method, the better flow and head values calculated by Gradient method iteration are directly calculated as opposed to correction values being calculated[8]. The head equations are written using the following format.

$$H_{x,k+1} - H_{y,k+1} - nK_i Q_{i,k+1}^{n-1}\ Q_{i,k+1} = (1-n)K_i Q_{i,k}^{n} \qquad (10)$$

Where K is Pipe resistance constant, i is Pipe number ,k is Iteration , Q is Discharge , n is Coefficient expressing the relationship between flow and head loss, H is Energy head , x is Node flow is leaving and y is Node flow is approaching. For all nodes and in pipes the network from figure 2, the flow equations at nodes 2, 3, 4, 5,6,7,8 and 9 are written using Kirchoff's first law and the head equations around loops 1, 2,3,4,5,6,7,8,9,10,11 and loop12 are written using the form of equation 10. Can be noted as following:

Node equations

Node2: $Q_1 - Q_5 - Q_2 = 0$
Node3: $Q5 - Q6=0.042$
Node4: $Q_4 - Q_3 - Q_{10} = 0$
Node5: $Q_2 + Q_3 - Q_7 - Q_8 = 0.023$
Node6: $Q_6 + Q_7 - Q_{11} = 0.108$
Node7: $Q_9 + Q_{10} = 0.087$
Node8: $Q_8 + Q_9 - Q_{12} = 0.088$
Node9: $Q_{11} + Q_{12} = 0.09$

pipe equations

Pipe1: $100 - H_2 - nK_1 Q_1^{n-1} = (1-n)K_1 Q_1^n$
Pipe2: $H_2 - H_5 - nK_2 Q_2^{n-1} = (1-n)K_2 Q_2^n$
Pipe3: $H_4 - H_5 - nK_3 Q_3^{n-1} = (1-n)\ K_3 Q_3^n$
Pipe4: $100 - H_4 - nK_4 Q_4^{n-1} = (1-n)K_4 Q_4^n$
Pipe5: $H_2 - H_3 - nK_5 Q_5^{n-1} = (1-n)K_5 Q_5^n$
Pipe6: $H_3 - H_6 - nK_6 Q_6^{n-1} = (1-n)K_6 Q_6^n$
Pipe7: $H_5 - H_6 - nK_7 Q_7^{n-1} = (1-n)\ K_7 Q_7^n$
Pipe8: $H_5 - H_8 - nK_8 Q_8^{n-1} = (1-n)K_8 Q_8^n$
Pipe 9: $H_2 - H_8 - nK_9 Q_9^{n-1} = (1-n)K_9 Q_9^n$
Pipe 10: $H_4 - H_7 - nK_{10} Q_{10}^{n-1} = (1-n)K_{10} Q_{10}^n$
Pipe 11: $H_6 - H_9 - nK_{11} Q_{11}^{n-1} = (1-n)K_{11} Q_{11}^n$
Pipe 12: $H_8 - H_9 - K_{12} Q_{12}^n\ n) -= (1 nK_{12} Q_{12}^{n-1}$

This system above is shown in equation (11) in the matrix format for the first iteration for the sample network.

$$\left[\begin{array}{cccccccccccccccccccc}
1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-nK_1Q_1^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -nK_2Q_2^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -nK_3Q_3^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -nK_4Q_4^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -nK_5Q_5^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -nK_6Q_6^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -nK_7Q_7^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -nK_8Q_8^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -nK_9Q_9^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -nK_{10}Q_{10}^{n-1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -nK_{11}Q_{11}^{n-1} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -nK_{12}Q_{12}^{n-1} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{array}\right]
\left[\begin{array}{c} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \\ H_7 \\ H_8 \\ H_9 \end{array}\right]
=
\left[\begin{array}{c} 0 \\ 0.042 \\ 0.108 \\ 0 \\ 0.023 \\ 0.087 \\ 0.088 \\ 0.09 \\ (1-n)K_1Q_1^n - H_1 \\ (1-n)K_2Q_2^n \\ (1-n)KQ_3^n \\ (1-n)K_4Q_4^n - H_1 \\ (1-n)K_5Q_5^n \\ (1-n)K_6Q_6^n \\ (1-n)K_7Q_7^n \\ (1-n)K_8Q_8^n \\ (1-n)K_9Q_9^n \\ (1-n)K_{10}Q_{10}^n \\ (1-n)K_{11}Q_{11}^n \\ (1-n)K_{12}Q_{12}^n \end{array}\right] \quad (11)$$

In the matrix of the relations (9) and (11) rows represent nodes and loops and columns represent pipes.

These relations are matrix reformulation of the second Kirchhoff's law. The sign for the term relates if the assumed flow is clockwise (1) or counter-clockwise (-1) relative to the loop.

3. Results and discussions

This paper compares the results of water distribution network model as described in figure 1for flow rates Q [m^3/s], and energy heads H [m], for three Q-based methods using Excel spreadsheet for carrying out analysis. They are Hardy Cross method, linear theory and Gradient method which are shown in Table (14). Carrying-out calculations until the corrections are less than or equal 0.001 m^3/s. Furthermore all flows have been calculated and the associated energy heads H [m], at any location.

Since, initial guesses are different for these methods, their number of iteration and central processing unit time cannot be technically compared each other. However, results were obtained using mathematical models developed utilizing spreadsheet on the basic the presentation presented in the form of graphs and patterns for two pipes of the network, pipe number 1&2, as example for comparison of convergence criterion, are showing comparison of the convergence performances for these methods, as well as iteration of Linear Theory was order of magnitudes larger than other data in Figures (3, 4), they were removed for better illustration, also number of iterations in each method at each pipe are tabled and summarized below. Iterations are continued until the flow values changes are small. Where the Linear Theory and Gradient Method results were approximately the same, except the difference in number of iterations, unlike Hardy Cross Method is exactly inefficient compared with both methods. On the other hand, Linear Theory and Gradient method are varying from hardy Cross method, and Gradient method vary from the Hardy Cross method because they take into account all loops simultaneously and therefore generally converge in less iteration. It should be mentioned that often a single iteration with any of these methods is more computationally intensive than a single Hardy Cross iteration. In applications to the node equations, however, convergence problems are possible, may result if poor initial conditions are selected (Jeppson, 1974)[1].

3.1. Hardy Cross Method (loop method)

First iteration for the water calculation for the network from figure 1 is shown in table 3. In table 3, loop and the pipes numbers are listed in the first and the second column, respectively. Pipe length and diameter expressed in meters is listed in the third and forth column, assumed water flow in each pipe expressed in m3/s is listed in the sixth column and calculated head looses in each pipe expressed in m is listed in the seventh column. The plus or minus preceding the flow, Q, indicates the direction of the pipe flow for the particular loop. A plus sign denotes clockwise flow in the pipe within the loop, a minus sign counterclockwise. All these assumption will not be changed.

Three iterations are enough for the calculation of water network from figure 1. Calculated flows for these first three iterations will be listed in table 4. The Δ Q values of the third correction are negative 0.001 m/s for loop 1 and loop 4 and 0.005 m3/s for loop 2 and loop 3. Also, calculated heads via flow rate for third iteration will be listed in Table 5.

Table 3: First correction results for water network from figure 1 –example, Flow in m^3/s.

1st Iteration

Loop	pipe	Dia [m]	L [m]	K_i	initial pipe flow assumptions Q [m^3/s]	Q_o^n [m^3/s]	h_L [m]	h_f/Q [m/m^3/s]	∑ h_L [m]	∑ h_f/Q [m/m^3/s]	ΔQ_j [m^3/s]	ΔQ [m^3/s]	Corrected Flow New Q [m^3/s]
1	AB	0.508	915	37.377	0.175	0.040	1.482	8.466				0.014	0.189
	BE	0.406	1220	148.449	0.045	0.003	0.476	10.571	-1.001	39.628	0.014	0.008	0.053
	EF	0.406	915	111.337	-0.088	-0.011	-1.235	14.039				-0.007	-0.095
	FA	0.61	1220	20.443	-0.263	-0.084	-1.723	6.552				0.014	-0.249
2	BC	0.508	915	37.377	0.130	0.023	0.854	6.572				0.006	0.136
	CD	0.406	1220	148.449	0.088	0.011	1.647	18.719	-0.813	79.536	0.006	0.006	0.094
	DE	0.305	915	448.354	-0.065	-0.006	-2.839	43.674				0.010	-0.055
	EB	0.406	1220	148.449	-0.045	-0.003	-0.476	10.571				-0.008	-0.053
3	FE	0.406	915	111.337	0.088	0.011	1.235	14.039				0.007	0.095
	EH	0.305	1220	597.805	0.045	0.003	1.916	42.569	-3.969	104.271	0.021	0.025	0.070
	HG	0.406	915	111.337	-0.088	-0.011	-1.235	14.039				0.021	-0.067
	GF	0.406	1220	148.449	-0.175	-0.040	-5.884	33.624				0.021	-0.154
	ED	0.305	915	448.354	0.065	0.006	2.839	43.674				-0.010	0.055
4	DI	0.305	1220	597.805	0.045	0.003	1.916	42.569	1.402	160.740	-0.005	-0.005	0.040
	IH	0.305	915	448.354	-0.045	-0.003	-1.437	31.927				-0.005	-0.050
	HE	0.305	1220	597.805	-0.045	-0.003	-1.916	42.569				-0.025	-0.070

Table 4. First three corrections for water network from Figure 1 – example.

Pipe number [ ]	Diameter (m)	Length (m)	Hazen-Williams Calculated resistance factor at C_{HW} =120 K_i	Flow in m^3/s Iteration 1	2	3	water velocity m/s
1	0.508	915	37.377	0.189	0.197	0.196	0.9675
2	0.406	1220	148.449	0.053	0.062	0.056	0.4328
3	0.406	915	111.337	0.095	0.085	0.091	0.7033
4	0.610	1220	20.443	0.249	0.241	0.242	0.8285
5	0.508	915	37.377	0.136	0.135	0.140	0.6911
6	0.406	1220	148.449	0.094	0.093	0.098	0.7574
7	0.305	915	448.354	0.055	0.063	0.057	0.7806
8	0.305	1220	597.805	0.070	0.061	0.061	0.8353
9	0.406	915	111.337	0.067	0.069	0.069	0.9449
10	0.406	1220	148.449	0.154	0.156	0.156	1.2056
11	0.305	1220	597.805	0.040	0.048	0.048	0.3710
12	0.305	915	448.354	0.050	0.042	0.042	0.5755

Table 5: Determined nodal heads results of third Correction for water network from figure 1 – example, Flow in m^3/s.

H1	100 Given at node number 1
No. node	energy heads at 3rd Iteration (m)
H2	98.16
H3	97.24
H4	98.53
H5	97.31
H6	95.41
H7	93.77
H8	93.96
H9	93.25

3.2. Linear theory method (*node method*)

First iteration for the water calculation for the network from figure 1 is shown in Table 6, 7. In Table 3, pipes numbers are listed in the first and the second column, respectively. Pipe length and diameter expressed in meters is listed in the third and forth column, assumed water flow in each pipe expressed in m^3/s is listed in the fifth column and calculated head looses in each pipe expressed in m is listed in the seventh column. The plus or minus preceding the flow, Q, indicates the direction of the pipe flow for the particular loop. A plus sign denotes clockwise flow in the pipe within the loop, a minus sign counterclockwise. All these assumption will not be changed.

Table 6: First correction results for water network from figure 1 – example, Flow in m^3/s.

					1st Iteration		
Pipe number [ ]		Dia (m)	L (m)	K_i	initial pipe flow assumptions Q [m3/s]	Q^n	h_L (m)
Q1	AB	0.508	915	37.377	0.438	0.217	8.102
Q2	BE	0.406	1220	148.449	0.438	0.217	32.180
Q3	EF	0.406	915	111.337	0.438	0.217	24.135
Q4	FA	0.61	1220	20.443	0.438	0.217	4.432
Q5	BC	0.508	915	37.377	0.438	0.217	8.102
Q6	CD	0.406	1220	148.449	0.438	0.217	32.180
Q7	DE	0.305	915	448.354	0.438	0.217	97.192
Q8	EH	0.305	1220	597.805	0.438	0.217	129.589
Q9	HG	0.305	915	448.354	0.438	0.217	97.192
Q10	GF	0.406	915	111.337	0.438	0.217	24.135
Q11	DI	0.406	1220	148.449	0.438	0.217	32.180
Q12	HI	0.305	1220	597.805	0.438	0.217	129.589

Twenty one iterations are enough for the calculation of water network from figure 1. Calculated flows for these first twenty one corrections will be listed in table 7. Also calculated heads via flow rate for twenty-first iteration will be listed in table 8.

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Table 7. First Twenty one corrections for water network from Figure 1 – example.

Pipe number [ ]	Diameter (m)	Length (m)	Hazen-Williams Factor at C_{HW} =120	Flow in m^3/s Iteration 1	2	3	4	5	6	7	8
1	0.508	915	37.377	0.438	0.219	0.198	0.215	0.201	0.213	0.202	0.211
2	0.406	1220	148.449	0.438	0.037	0.074	0.042	0.069	0.045	0.065	0.048
3	0.406	915	111.337	0.438	0.083	0.101	0.085	0.099	0.088	0.098	0.089
4	0.61	1220	20.443	0.438	0.219	0.240	0.223	0.237	0.225	0.236	0.227
5	0.508	915	37.377	0.438	0.182	0.125	0.174	0.132	0.168	0.137	0.163
6	0.406	1220	148.449	0.438	0.140	0.083	0.132	0.090	0.126	0.095	0.121
7	0.305	915	448.354	0.438	0.049	0.079	0.054	0.076	0.057	0.073	0.059
8	0.305	1220	597.805	0.438	0.047	0.073	0.051	0.069	0.053	0.067	0.055
9	0.406	915	111.337	0.438	0.050	0.051	0.050	0.051	0.050	0.051	0.051
10	0.406	1220	148.449	0.438	0.137	0.138	0.137	0.138	0.137	0.138	0.138
11	0.305	1220	597.805	0.438	0.081	0.054	0.077	0.057	0.074	0.060	0.072
12	0.305	915	448.354	0.438	0.009	0.036	0.013	0.033	0.016	0.030	0.018

Table 7. First Twenty one corrections for water network from Figure 1 – example, Flow in m^3/s.

Pipe number [ ]	Flow in m^3/s Iteration 9	10	11	12	13	14	15	16	17	18	19	20	21
1	0.203	0.210	0.204	0.209	0.205	0.208	0.206	0.208	0.206	0.208	0.206	0.207	0.2064
2	0.063	0.050	0.061	0.052	0.059	0.053	0.058	0.054	0.058	0.054	0.057	0.055	0.0567
3	0.096	0.090	0.095	0.091	0.095	0.092	0.094	0.092	0.094	0.092	0.094	0.092	0.0935
4	0.235	0.228	0.234	0.229	0.233	0.230	0.232	0.230	0.232	0.230	0.232	0.231	0.2316
5	0.141	0.160	0.144	0.157	0.146	0.156	0.147	0.154	0.148	0.154	0.149	0.153	0.1496
6	0.099	0.118	0.102	0.115	0.104	0.114	0.105	0.112	0.106	0.112	0.107	0.111	0.1076
7	0.071	0.061	0.069	0.062	0.068	0.063	0.068	0.064	0.067	0.064	0.067	0.065	0.0663
8	0.065	0.056	0.064	0.057	0.063	0.058	0.062	0.059	0.062	0.059	0.061	0.059	0.0610
9	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.0511
10	0.138	0.138	0.138	0.138	0.138	0.138	0.138	0.138	0.138	0.138	0.138	0.138	0.1381
11	0.062	0.071	0.063	0.070	0.064	0.069	0.065	0.068	0.065	0.068	0.066	0.067	0.0659
12	0.028	0.019	0.027	0.020	0.026	0.021	0.025	0.022	0.025	0.022	0.024	0.023	0.0241

Table 8: Determined nodal heads results of twenty-first correction for water network from figure 1 – example, Heads in m.

H1	100 Given at node number 1
No. node	energy heads at 21^{nd} Iteration (m)
H2	97.990
H3	96.881
H4	98.638
H5	97.260
H6	94.867
H7	95.791
H8	93.898
H9	93.296

3.3. Gradient Method (*node-loop method*)

First and second iteration for the water calculation for the network from figure 1 is shown in Table 9, 10. If sign of calculated flow is negative, this means that flow direction from previous iteration must be changed, otherwise, sing must be remained unchanged. In Table 9, 10, pipes numbers are listed in the first and the second column, respectively. Diameter and pipe length expressed in meters is listed in the third and forth column, and assumed water flow in each pipe expressed in m^3/s is listed in the fifth column for First iteration. The plus or minus preceding the flow, Q, indicates the direction of the pipe flow for the particular loop. A plus sign denotes clockwise flow in the pipe within the loop, a minus sign counterclockwise. All these assumption will not be changed. Also, corrected heads at second iteration is shown in table 11.

Table 9: First initial pipe flow values for water network from figure 1 – example, Flow in m^3/s.

1^{st} Iteration					
Pipe number [ ]		Dia (m)	L (m)	K_i	initial pipe flow assumptions Q [m3/s]
Q1	AB	0.508	915	37.377	0.438
Q2	BE	0.406	1220	148.449	0.438
Q3	EF	0.406	915	111.337	0.438
Q4	FA	0.61	1220	20.443	0.438
Q5	BC	0.508	915	37.377	0.438
Q6	CD	0.406	1220	148.449	0.438
Q7	DE	0.305	915	448.354	0.438
Q8	EH	0.305	1220	597.805	0.438
Q9	HG	0.305	915	448.354	0.438
Q10	GF	0.406	915	111.337	0.438
Q11	DI	0.406	1220	148.449	0.438
Q12	HI	0.305	1220	597.805	0.438

Table 10: second correction results for water network from figure 1 – example, Flow in m³/s.

2nd Iteration					
Pipe number [ ]		Dia (m)	L (m)	K	New pipe flow Q [m3/s]
Q1	AB	0.508	915	37.377	0.146
Q2	BE	0.406	1220	148.449	0.159
Q3	EF	0.406	915	111.337	0.110
Q4	FA	0.61	1220	20.443	0.291
Q5	BC	0.508	915	37.377	-0.012
Q6	CD	0.406	1220	148.449	-0.054
Q7	DE	0.305	915	448.354	0.112
Q8	EH	0.305	1220	597.805	0.134
Q9	HG	0.305	915	448.354	0.093
Q10	GF	0.406	915	111.337	0.180
Q11	DI	0.406	1220	148.449	-0.049
Q12	HI	0.305	1220	597.805	0.139

Table 11: Second correction results for water network from figure 1 – example , Heads in m.

No. node	**Corrected heads (m) at 2nd Iteration**
H2	101.87
H3	109.22
H4	98.32
H5	107.61
H6	144.08
H7	100.41
H8	144.6
H9	178.30

Sex iterations are enough for the calculation of water network from figure 1. Calculated flows for these first sex iterations will be listed in table 12. Also calculated heads for first sex iterations will be listed in table 13.

Table 12. First sex corrections results for water network from Figure 1 – example, Flow in m³/s.

Pipe number [ ]	**Diameter (m)**	**Length (m)**	**Hazen-Williams Factor at C_{HW} =120**	**Flow in m³/s Iteration**					
			K_i	1	2	3	4	5	6
1	0.508	915	37.377	0.438	0.147	0.207	0.207	0.207	0.207
2	0.406	1220	148.449	0.438	0.159	0.081	0.058	0.056	0.056
3	0.406	915	111.337	0.438	0.110	0.084	0.092	0.093	0.093
4	0.61	1220	20.443	0.438	0.291	0.231	0.231	0.231	0.231
5	0.508	915	37.377	0.438	-0.013	0.127	0.150	0.151	0.151
6	0.406	1220	148.449	0.438	-0.055	0.085	0.108	0.109	0.109
7	0.305	915	448.354	0.438	0.113	0.061	0.064	0.065	0.065
8	0.305	1220	597.805	0.438	0.134	0.081	0.063	0.060	0.060
9	0.406	915	111.337	0.438	0.094	0.060	0.052	0.051	0.051
10	0.406	1220	148.449	0.438	0.181	0.147	0.139	0.138	0.138
11	0.305	1220	597.805	0.438	-0.050	0.037	0.064	0.067	0.067

12	0.305	915	448.354	0.438	0.140	0.053	0.026	0.023	0.023

Table 13: First sex corrections results for water network from figure 1 – example, Heads in m.

H1	100 Given at node number 1					
No. node	Heads in m. Iteration					
	1	2	3	4	5	6
H2	101.880	98.113	97.971	97.980	97.980	97.98
H3	109.219	97.910	96.882	96.851	96.851	96.85
H4	98.316	98.721	98.649	98.644	98.644	98.64
H5	107.613	97.680	97.314	97.274	97.274	97.27
H6	144.085	96.538	94.573	94.398	94.396	94.39
H7	100.410	95.665	95.795	95.798	95.798	95.80
H8	144.596	93.826	93.986	93.981	93.981	93.98
H9	178.303	96.233	93.799	93.417	93.412	93.45

Table 14. Final Results of Analyzing the Sample Network

Hardy Cross				Linear theory				Gradient			
pipe	Q	node	H	pipe	Q	node	H	pipe	Q	node	H
AB	0.196	1	100	AB	0.2064	1	100	AB	0.207	1	100
BE	0.056	2	98.16	BE	0.0567	2	97.990	BE	0.056	2	97.98
EF	0.091	3	97.24	EF	0.0935	3	96.881	EF	0.093	3	96.85
FA	0.242	4	98.53	FA	0.2316	4	98.638	FA	0.231	4	98.64
BC	0.140	5	97.31	BC	0.1496	5	97.260	BC	0.151	5	97.27
CD	0.098	6	95.41	CD	0.1076	6	94.867	CD	0.109	6	94.39
DE	0.057	7	93.77	DE	0.0663	7	95.791	DE	0.065	7	95.80
EH	0.067	8	93.96	EH	0.0610	8	93.898	EH	0.060	8	93.98
HG	0.064	9	93.25	HG	0.0511	9	93.296	HG	0.051	9	93.45
GF	0.151			GF	0.1381			GF	0.138		
DI	0.047			DI	0.0659			DI	0.067		
IH	0.043			IH	0.0241			IH	0.023		
# of iteration		3		# of iteration		21		# of iteration			6

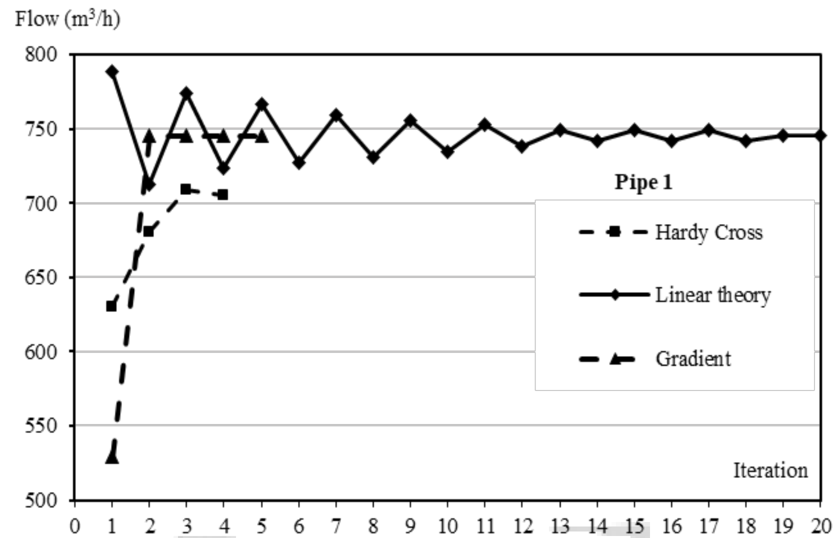

Figure 3. Comparison of the Convergence Performances for the Hardy Cross, Linear Theory and the Gradient Method.

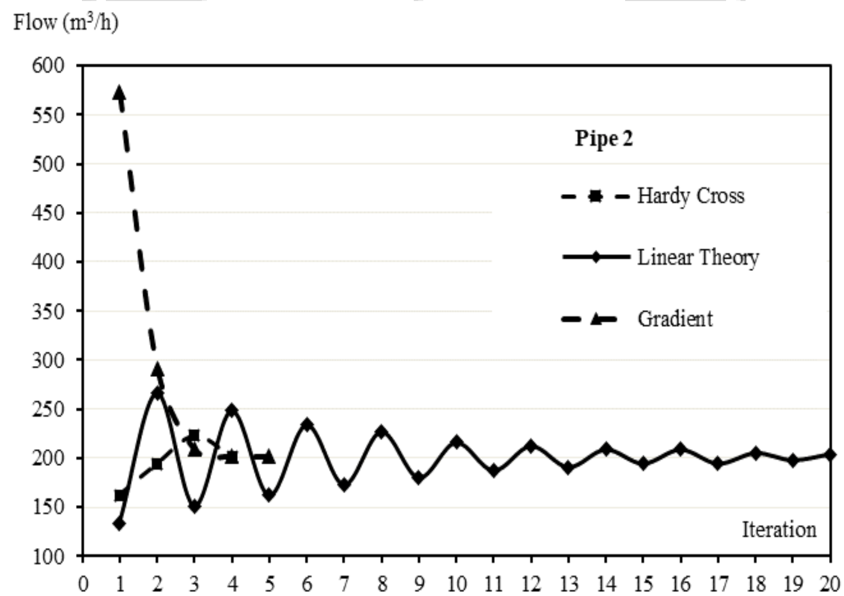

Figure 4. Comparison of the Convergence Performances for the Hardy Cross, Linear Theory and the Gradient Method.

4. Conclusions

In this paper, several methods have been presented to solve water distribution systems, these include Hardy Cross method, Theory Linear, and Gradient method. Through the analysis and comparison of the mathematical models and the results of these methods in terms of effectiveness of each method and the initial flow guesses, it can be noted that Hardy Cross method is in some instances may not converge if the initial guesses are too far off and especially, in large networks. Also must obey continuity of flow. Unlike the Theory Linear, and Gradient method the initial guesses of the flow do not have to adhere to continuity principles. In other words they do not require initialization of flows, besides always converges quickly. Moreover, it is characteristic of the gradient method over other methods that it can solve both of the looped and partly branched tube networks directly, and it is numerically more accurate and stable. Simulation or modeling of water distribution networks answers many questions about municipal and commercial water supply systems.

Most of the undergraduate engineering students study the Hardy Cross method of analysis, but it is not the current method used in the ready-made programs, in an attempt to help students better understand the ready-made programs in solving the water distribution system.

On the other hand, the aspect of developing and applying computational models for educational and practical purposes is one of the necessary skills to implement and analyze the behavior of numerical algorithms. This is important as it provides instructors with an alternative that allows the practice of algebraic and numerical methods for other courses. Microsoft Excel pathway or bridge enables students to analyze more realistic applications with the equivalent of manual solution development sufficient to learn basic engineering fundamentals.

5. References

[1] R.W.Jeppson, Analysis of flow in pipe networks. Sixth printing. Ann Arbor Science, Ann Arbor, Michigan, 1983.

[2] Prabhat K. Swamee Ashok K.Sharma. design of water supply pipe networks. Published by John Wiley & Sons, Inc., New Jersey, Published simultaneously in Canada 2008.

[3] Huddleston, D. H.; Alarcon, V. J.; Chen, W. Water Distribution Network Analysis Using Excel. Journal of Hydraulic Engineering 2004, 130, 1033–1035. https://doi.org/10.1061/(asce)0733-9429(2004)130:10(1033).

[4] Majid Niazkar, Seied Hosein Afzali. Analysis of Water Distribution Networks Using MATLAB and Excel Spreadsheet: Q-Based Methods, © 2017 Wiley Periodicals, Inc., Correspondence to S. H. Afzali (afzali@shirazu.ac.ir).
[5] David H. Huddleston, P.E., M.Asce1; Vladimir J. Alarcon2; And Wei Chen3, 2004, Water Distribution Network Analysis Using Excel, Journal Of Hydraulic Engineering © ASCE / October 2004.130:1033-1035.
[6] Selami Demir, Kaan Yetilmezsoy and Neslihan Manav, Development Of A Modified Hardy-Cross Algorithm For Time-Dependent Simulations Of Water Distribution Networks, PSP Volume 17 – No 8a. 2008.
[7] Dejan Brkić, Pavel Praks, An Efficient Iterative Method for Looped Pipe Network Hydraulics, Posted: 5 March 2019, doi:10.20944/preprints201903.0067.v1.
[8] Kevin Lansey, Larry W. Mays, Hydraulics of Water Distribution Systems chapter 4, Arizona State University.
[9] Lopes, A. M. G. Implementation of the Hardy-Cross Method for the Solution of Piping Networks. Computer Applications in Engineering Education 2004, 12, 117–125. https://doi.org/10.1002/cae.20006.
[10] T. Massoud and A. Zia, Dynamic management of water distribution networks based on hydraulic performance analysis of the system, Water Science and Technology: Water Supply Vol 3 No 1–2 pp 95–102 © 2003 IWA Publishing and the authors.
[11] John Wiley & Sons,Design Of Water Supply Pipe Networks, Inc., Hoboken, New Jersey 2008, online at http://www.wiley.com /go/permission.
[12] Sam C. M. Hui, Fluid Network Analysis II, Aug 2016, MEBS6008 Environmental Services II, The University of Hong Kong E-mail: cmhui@hku.hk.
[13] Todini, E. and Pilati, S. (1988) A gradient algorithm for the analysis of pipe networks. Computer applications in water supply. Wiley: Research Studies Press, 1–20.