Modeling of hydrodynamic pressure in a combined geometry pressure unit for wire drawing

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Abstract

In a conventional drawing process, the wire is pulled through a shaped die. Friction occurs between the products and the die which leads to the reduction in die life due to wear. There is a new technique using die-less for wire drawing and coating of wires or tubes. The wire submerged in polymer melt inside the pressure unit of different geometry, hydrodynamic pressure generated due to the flow of polymer melt between the wire and the pressure unit. In this research, a mathematical model was developed and theoretical study of the pressure distribution within the combined unit was completed through which a wire is drawn has been carried out. The unit used in this study consists of a parallel and exponential hyperbolic bore sections, having its smallest bore size greater than the incoming wire diameter. The unit is filled with a viscous fluid (polymer melt) through which the wire passes. The main objective of this project is to study theoretically the effect of changing dimensions on the pressure distribution along combined unit. Results have been presented and compared to other studies.

Keywords: *Hydrodynamic Pressure, Mathematical Model, Polymer Melt, Pressure Unit, Pressure Distribution*

1. Introduction

Traditionally, hydrodynamic means dynamics of fluids and under certain flow condition the hydrodynamic action generates very high pressure in the converging gap. This action is caused by existence of viscous fluid (like polymer melt) between two surfaces. The moving surface drags the fluid into the gap formed between it and the fixed surface and the relative motion between the moving surface and the fluid gives rise to the pressure [1]. The magnitudes of the hydrodynamic pressure are dependent on various parameters, such as the viscosity of the fluid, the geometrical shape of the surfaces as well as the relative speed between the moving and fixed surfaces [2]. A number of studies have been carried out applying this phenomenon to plastically reduce the diameter of wires and tubes by drawing through pressure units. The first attempt to employ hydrodynamic action was described by Christopherson and Naylor [3]. They employed a long tube with very close tolerances, attached to the front end of a conventional die. They used oil as a lubrication fluid. The use of a polymer melt as a lubricant in the drawing process was suggested by Symmons and Thompson [4,5]. They investigated the adherence of polymer coat and analyzed the criteria of polymer coat thickness on the steel wire. Further investigation was carried out by Stevens [6], who designed a hydrodynamic pressure wire coating unit consisting of a pressure tube connected to a conventional die. Crampton [7,8], carried out a study of the wire drawing using a unit similar to the one adopted by Stevens [6]. The apparatus Crampton and Stevens used consisted of a pressure tube connected to the forward end of a conventional die. The polymer melt was dragged into the tube by the motion of the wire generating high pressures which resulted in hydrodynamic lubrication and coating of the wire

2. Hydrodynamic Analysis

In order to study and verify the mechanics of the process of hydrodynamic pressure development within the unit, it is important to develop a suitable mathematical model. As a first step, such a model has been developed based on the assumption that the pressure medium demonstrates ideal Newtonian characteristics. The geometrical configuration of the combined pressure unit (parallel and exponential hyperbolic bore) and the wire during drawing are shown in Fig. (1).
To formulate the analysis, the following reasonable assumptions are made:

i. The fluid has the characteristics of a Newtonian, namely, the viscosity remains constant with shear rate and pressure.
ii. The flow of the fluid is purely axial and laminar.
iii. The thickness of the fluid layer is small compared to the bore of the unit.
iv. The pressure in the fluid is uniform in the thickness direction at any point along the length of the pressure unit.
v. The flow of the fluid is isothermal.
vi. The material of the wire is rigid

The analysis of the of flow is carried out in plane flow rather than in cylindrical ordinates

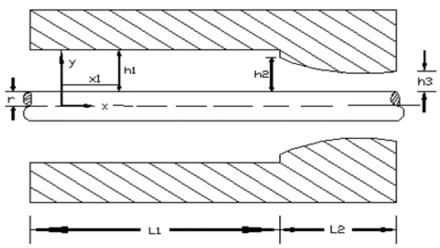

Fig. 1. Combined pressure unit (parallel and exponential bore)

2.1 *The first part of the unit*

The relationship between the pressure and shear stress gradient between the outer surface of the wire and the inner surface of the pressure unit is given by:

$$(\partial p/\partial x)_1=(\partial \tau/\partial Y)_1 \qquad (1)$$

The relationship between the shear stress and the rate of shear for the Newtonian fluid is given by:

$$\tau_1=\mu(\partial u_1/\partial Y)_1 \qquad (2)$$

Where μ is the viscosity and u_1 is the fluid velocity at a distance Y from the surface of the wire. Integrating equation (1) with respect to Y and noting that $(\partial p/\partial x)$ is constant with Y, we have:

$$\tau_1=p_1'Y+\tau c_1 \qquad (3)$$

Where $p' = (\partial p / \partial x)$ and τc_1 is the shear stress on the wire surface at Y = 0, substituting for τ_1 from equation (3) into equation (2) we have:

$$\mu (\partial u_1 / \partial Y) = p_1 ' Y + \tau c_1$$

Which after integration becomes

$$\mu u_1=(p_1'Y^2/2)+\tau c_1+C_1 \qquad (4)$$

$$u_1 = (p_1 ' Y^2 / 2\mu) + (\tau c_1 Y / \mu) + C_1 / \mu$$

Applying the boundary condition that at Y = 0 (at the surface of the wire) $u_1 = v_1$, we have:

$$C_1 = \mu v_1$$

So that

$$u_1=(p_1'Y^2/2\mu) +(\tau c_1Y/\mu) +v_1 \qquad (5)$$

Applying the boundary condition that at $Y = h_1$ (at the surface of the unit) $u_1 = 0$ in the equation (5) and rearranging we have:

$$\tau c_1h_1 / \mu = - (p_1'h_1^2 / 2\mu) - v_1$$

$$\tau c_1h_1 = - (p_1'h_1^2 / 2) - \mu v_1$$

$$\tau c_1=-(p_1'h_1/2) -(\mu v_1/h_1) \qquad (6)$$

The flow of the pressure medium in the first part of the unit Q_1, is given by:

$$Q_1 = \int_0^{h1} u_1 \, dY$$

Which upon substituting for $\mathbf{u_1}$ from equation (5) we get:

$$Q_1 = \int_0^{h1} (p_1'Y^2 / 2\mu + \tau c_1Y / \mu + v_1) \, dY$$

The above, after integrating becomes

$$Q_1 = p_1'h_1^3 / 6\mu + \tau c_1h_1^2 / 2\mu + v_1h_1$$

Substituting for $\boldsymbol{\tau c_1}$ from equation (6) into the above equation we get:

$$Q_1 = p_1'h_1^3 / 6\mu + [- p_1'h_1 / 2 - \mu v_1 / h_1] h_1^2 / 2\mu + v_1h_1$$

$$Q_1 = p_1'h_1^3 / 6\mu - p_1'h_1^3 / 4\mu - v_1h_1^2\mu / 2\mu h_1 + v_1h_1$$

$$Q_1 = [(2p_1'h_1^3 - 3p_1'h_1^3) / 12\mu] - [(v_1h_1 + 2v_1h_1) / 2]$$

$$Q_1=-p_1'h_1^3/12\mu+v_1h_1/2 \qquad (7)$$

Where $p_1' = (\partial p / \partial x)_1$

The continuity of fluid flow gives:

$$(\partial Q / \partial X) + (\partial Q / \partial Y) + (\partial Q / \partial Z) = 0,$$

but $\partial Q / \partial Y = \partial Q / \partial Z = 0$, and hence $\partial Q / \partial X = 0$

Therefore, for given h_1, h_2 and v_1

$\partial p / \partial x$ in equation (7) must be constant. Therefore $\partial p / \partial x = p_1' = Pms / L_1 =$ constant

Pms is the pressure at the step and L_1 is the length of the first part of the unit. It is shown that the pressure profile in the first part of the pressure unit is linear.

The pressure at any point $X_1 < L_1$ is given by:

$$P=\int_0^{x1} P_1' dX=(Pms/L_1)X_1 \qquad (8)$$

And P = Pms, when $X_1 = L_1$

$$Pms= \left\{\left[\left(\frac{6\mu v h1}{2B}\right)\left(\frac{1}{h3^2}-\frac{1}{h2^2}\right)\right]+\left(\frac{6\mu v}{B}\right)\left(\frac{1}{h2}-\frac{1}{h3}\right)\right\} / \left\{1+\left(\frac{h1^3}{2BL1}\right)\left(\frac{1}{h3^2}-\frac{1}{h2^2}\right)\right\} \qquad (9)$$

Where : $B =\frac{h2-h3}{L2}$

$$Pmax= \left(\frac{6\mu v}{B}\right)\left(\frac{1}{2\bar{h}}-\frac{1}{h3}-\frac{\bar{h}}{2h3^2}\right) \qquad (10)$$

$$Where: \bar{h} = h1 - \left(\frac{h1^3 * Pms}{6\mu v L1}\right) \qquad (11)$$

The axial stress on the wire at any point distance X_1 from the entry can be obtained by considering the shear force action on the surface of the wire,

Thus,

$$\sigma x_1 = \tau c_1x_1 / D$$

Substituting τc_1 from equation (6), we get:

$$\sigma x_1 = [(-p_1'h_1 / 2) - (v_1\mu / h_1)] x_1 / D$$

$$\sigma x_1 = (-p_1'h_1x_1 / 2D) - (\mu v_1x_1 / h_1D)$$

$$\sigma x_1 = (- Pms\, x_1h_1 / 2DL_1) - (\mu v_1x_1 / Dh_1)$$

$$\sigma x_1=(-x_1/D)[(Pmsh_1/2L_1)+(\mu v_1 / h_1)] \qquad (12)$$

2.2 *For the second part (exponential unit)*

The geometry defined for exponential unit is

$h(x) = a^2(x+b)^2 +c^2$.

The expressions for H(x) and G(x) in terms of h(x) is :

$$H(x) = \int_0^h \frac{1}{h^2} \, dx \ , \qquad G(x) = \int_0^h \frac{1}{h^3} \, dx$$

Substituting h(x) in the expression for H(x) and G(x), it becomes,

$$H(x)=\int_0^x \frac{1}{(a^2(x+b)^2+c^2)^2} dx \ ,$$

$$G(x) = \int_0^x \frac{1}{(a^2(x+b)^2+c^2)^3} dx$$

After integration we get,

$$H(x)= \left[\frac{1}{ac}\left(\arctan\left((b+x)\frac{a}{c}\right) - \arctan\left(\frac{ab}{c}\right)\right) + \left(\frac{b+x}{a^2(b+x)^2+c^2} - \frac{b}{a^2b^2+c^2}\right)\right] \quad (13)$$

and

$$G(x)=\frac{1}{4c^2}\left[\frac{b+x}{(a^2(b+x)^2+c^2)^2} - \frac{b}{(a^2b^2+c^2)^2}\right] + \frac{3}{4c^2}H(x) \quad (14)$$

The boundary conditions are:
(a) at $x = 0$, $P = 0$, and
(b) at $x = L$, $P = 0$
Therefore, with the boundary condition (a), the pressure expression becomes,

$$P(x) = 6\mu v[\, H(x) - h_b\, G(x)\,]$$

With the boundary condition (b), the position of optimum pressure is:

$$h_b = H(L) / G(L)$$

Therefore, the pressure profile in this unit is:

$$P(x) = 6\mu v\left[\, H(X) - \frac{H(L)G(X)}{G(L)}\,\right] \quad (15)$$

For exponential unit, the geometry can be presented as $h(x) = ((h_2-h_3) / L^2)(x-L)^2 + h_3$, therefore in this case the terms a can be substituted for $(h_2 - h_3)^{1/2} / L$, b for $-L$ and c for $(h_3)^{1/2}$.
Substituting the values for a, b and c at $x = L$, the expressions for H(L) and G(L) becomes,

$$H(L)= \frac{L}{2h_3}\left[\frac{1}{(\sqrt{h_3})(\sqrt{h_2-h_3})}\arctan\frac{\sqrt{h_2-h_3}}{\sqrt{h_3}} + \frac{1}{h_2}\right] \quad (16)$$

$$G(L)= \frac{1}{4h_3}\left[\frac{L}{(h_2)^2} + 3H(L)\right] \quad (17)$$

Where $L = L_2$
And with combined pressure unit (parallel & exponential bore):
At $x_2 = 0$, $P_2 = Pms$ and $h = h_2$ and at $x_2 = L_2$, $P_2 = 0$ and $h = h_3$

3. Results

Figure (2) shows the pressure distribution for gap ratios of h2/h3=10 and h1/h3=20 at three different drawing speeds.

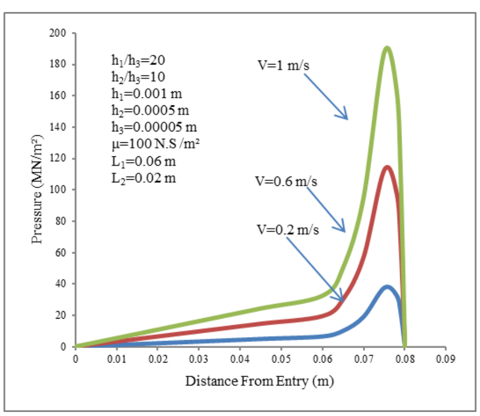

Fig. 2 Theoretical results of pressure distribution

This figure shows that the pressure increases gradually up to maximum value (at x= 0.075 m for V= 0.2 ,0.6 ,1 m/s) and then decreases gradually up, and the pressure increases as the drawing speed is increased. The maximum pressure was found to vary between (37 - 190) MN/m². Fig. (3) shows the pressure distribution of three different gap ratios (h_2/h_3 =12 , h_2/h_3 =16, h_2/h_3 =20)(h_3 is constant = 0.00005 m) at the drawing speed of 0.1 m/s. In this figure the maximum pressure increases as the gap ratio is decreased, and at gap ratio h_2/h_3 =12, the maximum pressure occurs at (x=0.075 m), whilst at gap ratio h_2/h_3 =16&20, the maximum pressure occurs at (x=0.078 m) and also in the first part of the unit , this changing of gap ratio h_2/h_3 does not significant affect on the pressure distribution. The maximum pressure was found to vary between (14 - 17) MN/m².

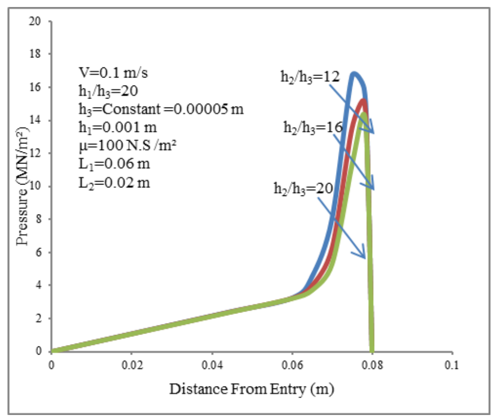

Fig. 3 Theoretical effect of gap ratio h2/h3 on pressure distribution

Fig. (4) shows the pressure distribution for two different length ratios (L1/L2 = 4, L1/L2 = 4.25) at the drawing speed of 0.1 m/s, where L2 is constant (L2 = 0.02 m).This figure shows that at V = 0.1 m/s, there is a slight change in the pressure distribution when L1/L2 was increased with L2 being kept constant (19-20 MN/m²).

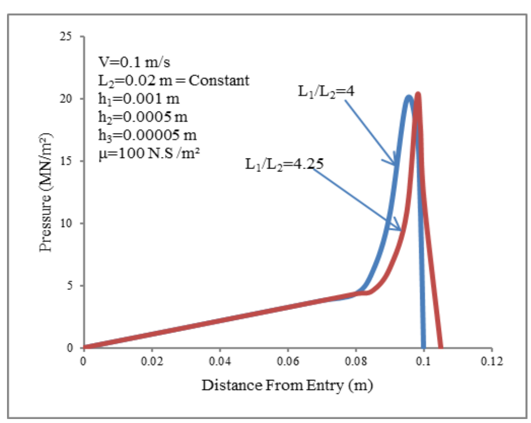

Fig. 4 Theoretical effect of length ratio L1/L2 on pressure distribution

4. Conclusion

In the present work, a theoretical analysis based on the Newtonian fluid characteristics has been achieved. The pressure unit in the present work consisted of combined parallel and exponential hyperbolic bore, having its smallest bore size greater than the incoming wire diameter, and large magnitude of the hydro-dynamic pressure is advantageous in obtaining greater information. The unit is filled with viscous fluid (polymer melt) through which the wire passes. The relation between pressure distribution in the unit and the distances from the outside diameter of the wire to the inlet surface of the die are studied. The pressure increases as the drawing speed is increased for different gap ratios. The maximum pressure increases as the gap ratio is decreased at the same drawing speed. The affects of changing lengths in the pressure unit has been achieved. The increase of the first part length in the pressure unit did not affect maximum pressure for a given drawing speed.

Nomenclature

p'	Pressure gradient in the pressure unit
τ	Shear stress in the fluid in the pressure unit
τc	Shear stress on the wire in the pressure unit
σx	Axial stress in the wire
Pms	Pressure at the step
Pmax	Maximum pressure in the pressure unit
Q	Flow of the fluid in the pressure unit
u	Velocity of the fluid
V	Velocity of the wire
h_1	Inlet gap in the first part of the unit
h_2	Middle gap at the step of the unit
h_3	Exit gap in the second part of the unit
L_1	Length of the first part of the pressure unit
L_2	Length of the second part of the pressure unit
μ	Viscosity of polymer melt
h_b	The position of optimum pressure

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