Non- Newtonian Hydro-dynamic pressure distribution in a complex geometry coating unit (stepped & converging exponential part) Theoretical Predictions.

M. A. Nwir, Ahmed L. Ali, and H. S. Elhaji

College of Engineering Technology, Janzour, Libya.

Abstract

Hydrodynamic coating of wires and wire robs is a process, whereby a metal continuum is coated by passing it through a hydrodynamic pressure unit filled with polymer melt. The geometry of the coating unit is parallel (stepped) in the first section while becomes converging exponential the second part. Theoretical analysis has been found in literature and applied to predict the pressure distribution in the complex pressure unit. In this paper a mathematical model has been developed for the pressure distribution within a complex geometry pressure unit for hydrodynamic coating of wires and wire robs. Theoretical results are obtained for different continuum speeds in terms of the changes in viscosity, shear stress and pressure distributions within the unit, such results will help in assessing and evaluating the performance of such coating units

Keywords: *coating units , hydrodynamic coating , plasto-hydrodynamic pressure , pressure distribution , polymer melt , mathematical modeling , converging exponential , viscosity , shear stress.*

1. Introduction

In plasto-hydrodynamic die-less wire drawing process, the conventional reduction die is replaced by a pressure unit of certain internal geometry. The deformation is induced by the combined effect of the hydro-dynamic pressure and stress generated in the unit due to the motion of the wire, the viscous action of the polymer melt and the pulling action of the wire. The dimensions of the unit are such that the smallest bore size is greater than the incoming wire diameter. In this system, larger magnitude of the hydro-dynamic pressure is advantageous in obtaining greater deformation. A different studies has been carried out by different researchers on different types of pressure units such as the complex pressure unit, the parallel bore pressure unit, the simple tapered pressure unit were the effect of different parameters affected the process studded for example the drawing force, the units internal dimensions, the assumption of non-Newtonian pressure fluid was also verified, those studies published on different journal and conferences.

Pressure effect on viscosity:

The viscosity of a fluid determines by the free volume several theories suggestions [7,8]. The free volume of a fluid is the difference between the actual volume and a volume in which such close packaging of the molecules occurs that no motion can take place. The greater the free volume the easier it is for flow to take place. At very high pressures no free volume is left in the liquid which becomes solid [6]. The viscosity of liquids increases, when pressure increases because of distance change between molecules [6].

Critical Shear stress

The critical shear stress is the stress at which the uniformity of the non-Newtonian fluids such as polymer melt flow ceases to exist [6]. There is a general agreement on the following points:

- The critical shear stress is independent of the die length and diameter.
- The critical shear stress is in the region of (0.1 - 1.0 MN/m^2) for most polymers.
- A discontinuity in the slope of viscosity - shear stress occurs.
- The flow defect is often associated with die entrance and the surface finish.
- The critical shear stress does not vary widely with temperature [5].

2. Theoretical Analysis

Following analysis based on the geometrical configuration shown in Figure (1). To formulate the analysis the following assumptions were made:

- The flow of polymer melt is axial and laminar.
- The thickness of the polymer melt layer is small compared to the unit dimensions.
- The shear stress in the polymer melt is assumed to be constant for a certain drawing speed.

- The fluid (pressure medium) has the characteristics of a non-Newtonian fluid, namely, the viscosity is dependent on the shear rate and pressure.

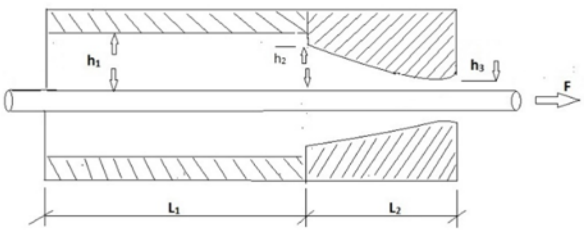

Figure (1) Complex Geometry Pressure Unit

Determination of the pressure within the complex unit

It is shown that the pressure profile in the second part of the unit is not linear. It is known that the viscosity of polymer melt is dependent on the pressure and this effect should be taken into account in the analysis for satisfactory solution. With reference to Figure (1) a generalized equation relating viscosity and pressure may be shown to take the form,

$$\mu = \mu_0 + \frac{[a + bP_{av}^2]}{\gamma}$$

where **a** and **b** are constants and μ_0 is the initial viscosity of polymer melt at ambient pressure, $\acute{Y}$ is the apparent shear rate and is given by,

$$\gamma = \frac{V}{h_a}$$

where V is the continuum velocity and $h_a=(h_2+h_3)/2$, is the average converging exponential gap thickness.

1- <u>Plasto-hydrodynamic pressure model within the parallel part of the unit.</u>

The relationship between the pressure and the shear stress gradient in the parallel part of the unit is given by,

$$\left(\frac{dp}{dx}\right) = \left(\frac{d\tau_1}{dy}\right) \qquad (1)$$

An empirical equation has been proposed by RABINOWITSCH for polymer melt relating shear stress to shear rate,

$$\tau_1 + k\tau_1^3 = \mu\left(\frac{du}{dy}\right) \qquad (2)$$

Considering Figure (1), the steady state flow of polymer is given by equation (1) in which after integration w.r.to y gives:

$$\left(\frac{dp_1}{dx}\right)y + C_1 = \tau_1 \qquad (3)$$

Where C_1 is a constant.

At the surface of the wire $y = 0$ and $\tau_1 = \tau_{c1}$ so that,

$$\left(\frac{dp_1}{dx}\right)y + \tau_{c1} = \tau_1 \qquad (4)$$

Noting that $\tau = \tau_1$ in equation (3) and combining equations (3) and (4) gives,

$$\mu\left(\frac{du_1}{dy}\right) = \left(\frac{dp_1}{dx}\right)y + \tau_{c1} + k\left[\left(\frac{dp_1}{dx}\right)y + \tau_{c1}\right]^3$$

Integration of the above equation w.r.to y gives,

$$\mu u_1 = \left(\frac{dp_1}{dx}\right)\frac{y^2}{2} + \tau_{c1}y + k\left[\left(\frac{dp_1}{dx}\right)^3\frac{y^4}{4} + \left(\frac{dp_1}{dx}\right)^2 y^3\tau_{c1} + \left(\frac{dp_1}{dx}\right)\frac{3y^2\tau_{c1}^2}{2} + \tau_{c1}^3 y\right] + C_2 \qquad (5)$$

Where C_2 is constant,

The boundary conditions at speeds at which slip does not occur are,

(1)- at wire surface $y = 0$, $u_1 = v$

(2)- at the unit surface $y = h_1$, $u_1 = 0$

Where V is the velocity of the undeformed wire. Substituting condition, (1) into the above equation,

$$V = C_2/\mu$$

So that,

$$u_1 = \left(\frac{dp_1}{dx}\right)\frac{y^2}{2\mu} + \frac{\tau_{c1}}{\mu}y + \frac{k}{\mu}\left[\left(\frac{dp_1}{dx}\right)^3\frac{y^4}{4} + \left(\frac{dp_1}{dx}\right)^2 y^3\tau_{c1} + \left(\frac{dp_1}{dx}\right)\frac{3y^2\tau_{c1}^2}{2} + \tau_{c1}^3 y\right] + V \qquad (6)$$

Applying the boundary conditions (2) to find τ_{c1} at the surface of the pressure unit,

$$0 = (\frac{dp_1}{dx})\frac{h_1^2}{2\mu} + \frac{\tau_{c1} h_1}{\mu} + \frac{k}{\mu}[(\frac{dp_1}{dx})^3 \frac{h_1^4}{4} + (\frac{dp_1}{dx})^2 h_1^3 \tau_{c1} + (\frac{dp_1}{dx})\frac{3h_1^2\tau_{c1}^2}{2} + \tau_{c1}^3 h_1] + V$$

Rearranging the above equation in terms of the power of τ_{c1}

$$\tau_{c1}^3 + (\frac{dp_1}{dx})\frac{3h_1\tau_{c1}^2}{2} + \tau_{c1}[\frac{1}{k} + (\frac{dp_1}{dx})^2 h_1^2] + [(\frac{dp_1}{dx})\frac{h_1}{2k} + (\frac{dp_1}{dx})^3\frac{h_1^3}{4} + \frac{\mu v}{kh_1}] = 0$$

After solving

$$\tau_{c1} = (-\frac{\mu v}{2kh_1} + [\frac{\mu^2 v^2}{4k^2 h_1^2} + \frac{1}{27}(\frac{1}{k} + (\frac{dp_1}{dx})^2\frac{h_1^2}{4})^3]^{\frac{1}{2}})^{\frac{1}{3}} + (-\frac{\mu v}{2kh_1} - [\frac{\mu^2 v^2}{4k^2 h_1^2} + \frac{1}{27}(\frac{1}{k} + (\frac{dp_1}{dx})^2\frac{h_1^2}{4})^3]^{\frac{1}{2}})^{\frac{1}{3}} - \frac{h_1}{2}(\frac{dp_1}{dx}) \quad (7)$$

The above equation gives the shear stress on the wire before deformation for known values of (dp/dx). The flow of liquid polymer in the axial direction within the gap before the step may be given by,

$$Q_1 = \int_0^{h_1} u_1\, dy \quad (8)$$

Substituting for U_1 into the above equation and integrating,

$$Q_1 = (\frac{dp_1}{dx})\frac{h_1^3}{6\mu} + \frac{\tau_{c1}h_1^2}{2\mu} + \frac{k}{\mu}[(\frac{dp_1}{dx})^3\frac{h_1^5}{20} + (\frac{dp_1}{dx})^2\frac{h_1^4\tau_{c1}}{4} + (\frac{dp_1}{dx})\frac{h_1^3\tau_{c1}^2}{2} + \frac{\tau_{c1}^3 h_1^2}{2}] + vh_1 \quad (9)$$

The flow of liquid polymer obey the continuity equation so that,

$$\frac{dQ_x}{dx} + \frac{dQ_y}{dy} + \frac{dQ_z}{dz} = 0$$

Under axial and laminar flow conditions,

$$\frac{dQ_y}{dy} = \frac{dQ_z}{dz} = 0$$

And hence,

$$\frac{dQ_x}{dx} = 0$$

Substituting for τ_{c1} from equation (7) into equation (9) and nothing that h_1 ,μ and V are constants and that $(dQ_x/dx)=0$, it is shown that,

$$(\frac{dp_1}{dx}) = \frac{P_{step}}{L_1} = Constant \quad (10)$$

Where P_{step} is the pressure at the step and L_1 is the length of the parallel part of the unit. Thus the pressure profile in the parallel part of the unit is linear. However, P_{step} cannot be determined at this stage since equation (9) contains the unknown variable Q_1 and it must be defined.

2-Plasto-hydrodynamic pressure model within the Converging Exponential part:

Since the flow of the pressure fluid within the parallel part of the unit is Q_1 which is the same as the flow within the converging exponential part and equals to Q_2 and regarding the continuity equation so,

Equation (9) can be written as

$$Q = -\frac{(\frac{dp}{dx})h^3}{12\mu} - \frac{k}{\mu}\left(\frac{\tau^2(\frac{dp}{dx})h^3}{4}\right) + \frac{Vh}{2} \quad (11)$$

Considering a position where $h=h_b$ ie, the pressure is maximum. Therefore in that position (dp/dx)=0 which gives $Q=(Vh_b)/2$

Thus,

$$\frac{dp}{dx} = \frac{6\mu v}{[1+3k\tau^2]}[\frac{1}{h^2} - \frac{h_b}{h^3}]$$

Let,

$$H(x) = \int_0^x \frac{1}{h^2}dx , \qquad G(x) = \int_0^h \frac{1}{h^3}dx$$

The boundary conditions are,

(1)- at the entry of the unit X=0, P=0

(2)- at the exit of the unit X=L, P=0

Therefore, with the boundary condition (1) the pressure expression becomes

$$P(x) = \frac{6\mu v}{[1+3k\tau^2]}[H(x) - h_b G(x)] \quad (12)$$

With the boundary condition (2) the position of optimum pressure is

$$h_b = H(L) / G(L)$$

Therefore, the pressure profile in this part is

$$P(x) = \frac{6\mu v}{[1+3k\tau^2]}[H(x) - \frac{H(L)G(x)}{G(L)}] \quad (13)$$

For Converging Exponential shape part of the unit, the geometry can be considered as ,

$$h(x) = a^2 (x+b)^2 + c^2$$

The expression for H(x), and G(x) in terms of h(x) becomes,

$$H(x) = \int_0^x \frac{1}{(a^2(x+b)^2 + c^2)^2}\, dx$$

After integration it becomes

$$H(x) = \frac{1}{2C^2}\left[\frac{1}{ac}\left(\arctan(b+X)\frac{a}{c}\right) - \arctan\left(\frac{ab}{c}\right)\right] + \left[\frac{b+X}{a^2(b+x)^2+c^2} - \frac{b}{a^2b^2+c^2}\right] \quad (14)$$

And in the same way substituting h(x) in the expression for G(x), it gives

$$G(x) = \int_0^x \frac{1}{(a^2(x+b)^2 + C^2)^3}$$

After integration it gives,

$$G(x) = \frac{1}{4c^2}\left[\frac{b+x}{(a^2(b+x)^2+c^2} - \frac{b}{(a^2b^2+c^2)^2}\right]\frac{3}{4c^2}H(x) \quad (15)$$

Where, <u>d = c/a</u>

For plasto-hydrodynamic converging exponential unit , the geometry can be presented as :

$$h(x) = \left[\frac{(h_1-h_2)}{L^2}\right](x-L)^2 + h_2 \quad (16)$$

therefore, in this case substitution for

a $\rightarrow$ **$(h_1 - h_2)^{1/2}$ /L, b** $\rightarrow$ **-L , C** $\rightarrow$ **$h_2^{1/2}$**

substituting the values for **a, b** and **c** at x=L , the expression for G(L), and H(L) becomes

$$G(L) = \frac{1}{4h_2}\left[\frac{L}{h_1^2} + 3H(L)\right] \quad (17)$$

and H(L) becomes

$$H(L) = \frac{L}{2h_2}\left[\frac{1}{\sqrt{h_2}\sqrt{h_1-h_2}}\arctan\left(\frac{\sqrt{h_1-h_2}}{\sqrt{h_2}}\right) + \frac{1}{h_1}\right] \quad (18)$$

3. Results & Discussion

Simulation has been carried out as to the mathematical model, results obtained demonstrated in the following figures. Fig (2) Demonstrates the effect of drawing speed on pressure distribution within the unit speed varies from 0.4 m/sec to 2 m/sec , as speed increases at 2 m/sec a maximum pressure of 360 bar was obtained whereas at 1 m/sec the maximum pressure was

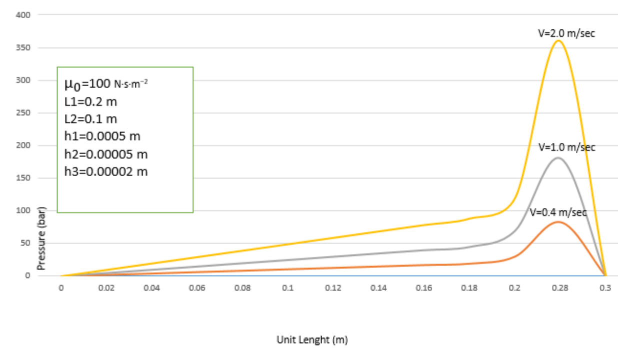

Fig. 2, effect of Drawing Speed on Pressure Distribution within the Unit

Figure (3) shows the effect of outlet gap (h_3) on pressure distribution within the unit and the maximum pressure gained , it could be observed that as (h_3) becomes smaller pressure generated for the same drawing speed is higher so for (h_3)= 0.00001 m a maximum pressure of 300 bar is gained for drawing speed of 0.4 m/sec whereas for the speed and outlet gap of (h_3)= 0.00003 m the maximum pressure was 160 bar , so the figure gives the idea of smaller the outlet gap the higher the pressure generated within the unit.

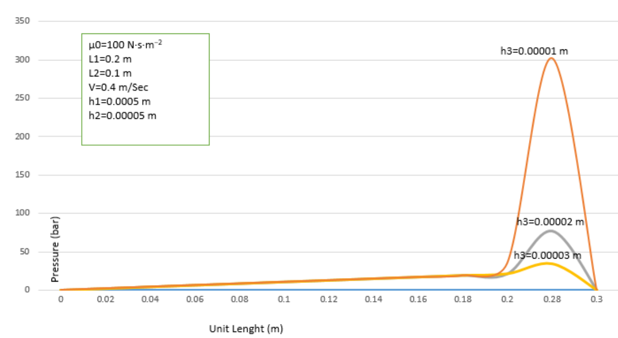

Fig. 3, effect of outlet gap (h3) on pressure distribution within the unit

Figure (4) Demonstrates the effect of μ_0 the initial viscosity on pressure distribution and maximum pressure generated within the unit regarding the data given in the Box, as μ_0 increases higher pressure generated within the unit so for initial viscosity of 200 N.S.m^{-2} the maximum pressure was 150 bar where as for initial viscosity of 100 N.S.m^{-2} the maximum pressure was 80 bar. For the given data within the box in the figure (5), the figure shows the effect of the first part of the unit on the pressure distribution and maximum pressure gained as L_1changes the maximum pressure within the second part of the unit changes positively so for L_1 of 0.2 m the maximum pressure gained was 70 bar were as when L_2 of 0.14 the maximum pressure generated was 25 bars, as L_2 increases the higher pressure generated.

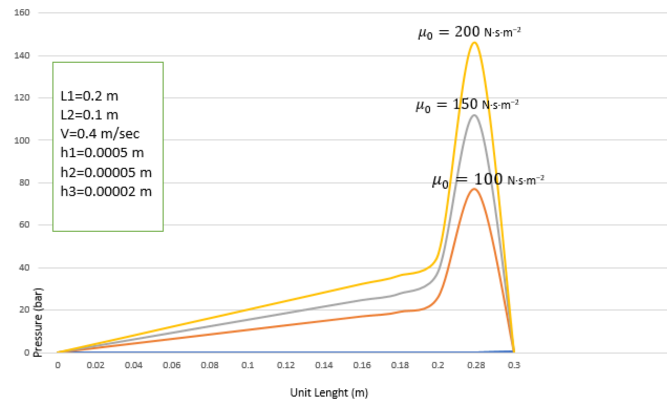

Fig. 4, effect change μ0 (initial viscosity) on pressure distribution and maximum pressure within the unit

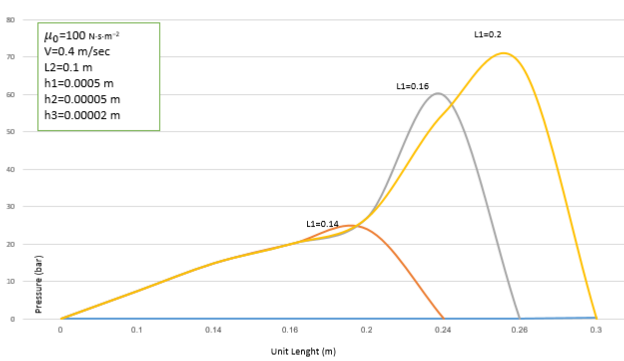

Fig. 5, effect change of L1 on pressure distribution and maximum pressure within the unit

4. Conclusion

A mathematical model has been developed for a combined geometry pressure unit assuming non-Newtonian pressure medium, results was discussed which should use in comparing coating and pressure units for coating and drawing wires and wire robs.

5. References

1- S. Akter, "Study of Viscous Flow During Thin Film Polymer Coating and Drawing of Continuum". PhD Theses, Dublin City University, Ireland, (1997).

2- Al-Natour, "Plasto-Hydrodynamic Pressure due to the Flow of Viscous Fluid through A Confined Passage", M.Eng Thesis, Dublin City University, (1989).

3- D.G. Christopherson And P.B. Naylor, "Promotion of Fluid lubrication in Wire Drawing", Proc. Inst. Mech. Eng., 643 , (1955).

4- P.J. Thompson And G.R. Symmons, "A Plasto-Hydrodynamic Analysis of The Lubrication and Coating of Wire Using Polymer Melt during Drawing", Proc. Inst. Mech. Eng., 191, (13), 115, (1977).

5- G. R. Symmons, And P. J. Thompson, "Hydrodynamic Lubrication And Coating of Wire Using a Polymer Melt During The Drawing Operation", Wire Industry, Vol-45, 469-473, 483, (1978).

6- A.J. Stevens, "A Plasto-Hydrodynamic Investigation of the Lubricatio and Coating of Wire Using a Polymer Melt During Drawin M. Phil. Thesis, Sheffield City Polytechnic, (1979).

7- R. Crampton, G .R. Symmons And M.S.J. Hashmi, " A Non- Newtonian Plasto-Hydrodynamic Analysis of the Lubrication and coating of Wire Using a Polymer Melt During Drawing", Proc. Int.Symposium On Metal Working Lubrication, Sanfrancisco, USA,(1980).

8- M. S. J. Hashmi, R. Crampton and G. R. Symmons, "Effects of Strain Hardening And Strain Rate Sensitivity of The Wire Material During Drawing Under Non-Newtonian Plasto-hydrodynamic Lubrication Conditions", International Journal Of Machine Tool Design Research, Vo l2, 1, P 71-86, (1981).