

A Numerical Solution for Plasto-hydrodynamic Die-Less Drawing of a Continuum Through a Simple Tapered Unit

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Abstract

In the conventional drawing, the wire diameter is reduced by pulling it through a reduction die of a tapered bore size. A process known as die-less wire drawing has been developed in which polymer melts were used as a pressure medium in a reduction unit of stepped-bore geometry.

The process involves pulling the continuum through a simple tapered shape die-less unit which is filled with a viscous fluid (polymer melt), the continuum outer diameter is smaller than the inlet and outlet gaps of the unit so no metal to metal contact taking place, the pulling action causes yielding of the continuum as the combination of pressure and drawing stress increases (Tresca hypothesis) and a reduction in area of the continuum is then obtained. Deformation of the wire is directly dependent on the drawing speed, being caused by the pressure generated and the shear stress developed in the polymer melt. A Newtonian behavior of the pressure medium (polymer melt) and a rigid non-linear strain-hardening continuum will be considered in which non-linear equations will be formulated for the deformation of the continuum through the unit.

1. Introduction

In conventional drawing of wire, the diameter is reduced by pulling it through a reduction die, resulting in wear and time dependent size of product [1,2,3]. This is done by pulling metal through a die by means of a tensile force applied to the exit side of the die. The reduction in diameter of a solid bar or rod by successive drawing is known as bar, rod, or wire drawing, depending on the diameter of the final product. In addition to direct application such as electrical wiring, wire is the starting material for many products including wire frame.

A developed technique enables the wire diameter to be reduced without using any conventional reduction die [4,5,6]. In this process the wire is pulled through a tubular orifice of tapered bore which is filled with a viscous fluid. An important feature of the process is that the smallest bore size of the orifice is always greater than the diameter of the unreformed wire and hence no metal to metal contact takes place. The pulling action of the wire through the viscous fluid gives rise to a drag force and generates hydrodynamic pressure on the wire. The magnitudes of the hydrodynamic pressure are dependent on various parameters, such as the viscosity of the fluid, the geometrical shape of the surfaces as well as the relative speed between the moving and fixed surfaces [6,7,8]. by design and optimization, the combined drag force and pressure initiates plastic yielding and permanent deformation to the wire.

Experimental work has shown that mild steel, copper and stainless steel wires can be drawn using either type of the units³, with polymer melt as the hydrodynamic pressure medium, and that products having comparable dimensional and surface qualities can be obtained [9,10].

2. Analysis

In order to establish a mathematical formulation of the process the following assumptions are made.

1. the thickness of the fluid layer is small compared to the bore of the orifice.
2. the pressure in the fluid is uniform in the thickness direction at any point along the length
3. The flow is in steady state.
4. The pressure gradient dP/dx is independent of y .

2.1 pressure -before deformation

The analysis is based can the geometrical configuration show in Fig.1(a) of the orifice and the continuum, the gap at any point is given by:

$$h = h_1 - BX \quad \text{Where,} \quad B = (h_1 - h_2) / L$$

The relationship between the pressure and shear stress gradient for a Newtonian fluid medium is given by

$$\left(\frac{dP}{dx}\right) = \left(\frac{d\tau}{dy}\right) \quad (1)$$

And the shear stress as

$$\tau = \mu \left(\frac{du}{dy}\right)^n \quad (2)$$

This equation is applicable for any type of fluid. Here, n is the power law index which equals to 1 for Newtonian fluid, greater than 1 for dilatants fluid and less than 1 for a pseudo plastic fluid. In this equation, (du/dy) is the shear rate. hence the velocity distribution in the gap is given by

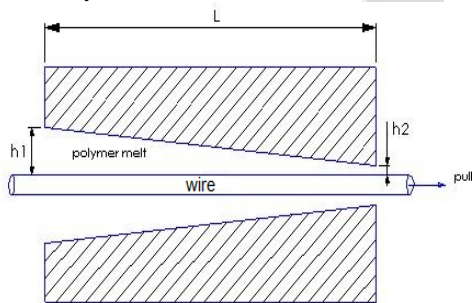
$$u = \frac{p'y^2}{2\mu} - \frac{p'h y}{2\mu} - \frac{Vy}{h} + V$$

The flow of the polymer melt in the unit is,

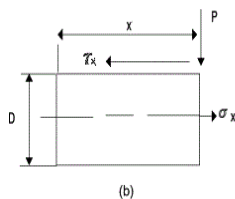
$$Q = \int_0^h u \, dy$$

The boundary conditions are

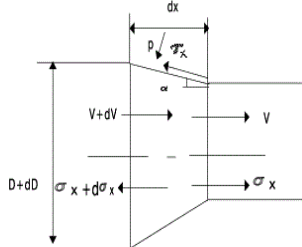
- (a) at $y=0$, $U = V$ at the wire surface
- (b) at $y=h$, $U = 0$ at the surface of the unit



(a)



(b)



(c)

Fig.1 (a) Schematic diagram showing the die-less reduction unit (b , c) Mode of deformation of a small element of the continuum

$$Q = - \frac{p'h^3}{12\mu} - \frac{k\tau^2 p'h^3}{4\mu} + \frac{Vh}{2}$$

For steady state conditions

$$Q = - \frac{p'h^3}{12\mu} + \frac{Vh}{2}$$

Now integrating it again and noting that $(dQ/dx) = 0$ it gives

$$\frac{p'h^3}{6\mu} = Vh + C_3$$

The optimum pressure condition is at $\{dp/dx\} = 0$, where $h = \bar{h}$

$$C_3 = - V \bar{h}$$

Substitute into the above equation and rearrange gives

$$p' = 6\mu V \left(\frac{1}{h^2} - \frac{\bar{h}}{h^3} \right) \quad (3)$$

The pressure at any point (x) within the orifice may be expressed by:

$$P = \frac{6\mu V}{B} \left\{ \frac{1}{(h_1 - BX)} - \frac{\bar{h}}{2(h_1 - BX)^2} - \frac{1}{h_1} + \frac{\bar{h}}{2h_1^2} \right\} \quad (4)$$

Where, (P) is the hydrodynamic pressure exerted on the continuum. However (\bar{h}) is still not determined. Using the boundary condition that $P = 0$ at $x = L$ where $(h_1 - BX) = h_2$ in equation (6) and re-arranging:

$$\bar{h} = \frac{2 h_1 h_2}{(h_1 + h_2)} \quad (5)$$

$$\bar{X} = \frac{h_1 L}{(h_1 + h_2)} \quad (6)$$

2.2 Axial stress -before deformation

An axial stress in the wire will be produced by the shear stresses in the fluid acting on the Surface of the wire.

$$\tau = \mu \left(\frac{du}{dy} \right)$$

The expression for the shear stress at any depth in the fluid may be obtained as:

$$\tau = \frac{1}{2} \left(\frac{dP}{dx} \right) (2y - h) - \frac{\mu V}{h}$$

at $y = 0$

$$\tau_x = - \frac{h}{2} \left(\frac{dP}{dx} \right) - \frac{\mu V}{h} \quad (7)$$

substituting for $\left(\frac{dP}{dx} \right)$ from equation (3), and $h = h_1 - BX$, into (7) and rearranging terms thus becomes

$$\tau_x = \mu V \left(\frac{3\bar{h}}{(h_1 - BX)^2} - \frac{4}{(h_1 - BX)} \right) \quad (8)$$

This shear stress gives rise to drag force on the continuum and at a point (x) within the orifice this may be expressed as:

$$F_d = \int_{x=0}^{x=l} \pi D_1 \tau_x dx$$

After integrating and noting that at $x = 0$, $F_d = 0$

$$F_d = \frac{\pi \mu V D_1}{B} \left(\frac{3\bar{h}}{(h_1 - BX)} - \frac{3\bar{h}}{h_1} + 4 \ln \left(\frac{h_1 - BX}{h_1} \right) \right) \quad (9)$$

The axial stress developed in the continuum is thus:

$$\sigma_x = \frac{4F_d}{\pi D_1^2} = \frac{4x\tau_x}{D_1} \quad (10)$$

3. Analysis at the deformation zone

3.1 Plastic yielding

The combined effects of the axial stress and the hydrodynamic pressure can cause plastic yielding of the wire at any point, x, within the orifice as soon as the plastic yield criteria becomes satisfied. If the material of the continuum is assumed to be rigid non linearly strain hardening then the flow stress can be.

$$Y = Y_0 + K_0 \varepsilon^n \quad (11)$$

Therefore yielding occurs as soon as $Y = Y_0$, so that at X_1 according to Tresca yield criterion

$$P + \sigma_x = Y_0 \quad (12)$$

given values of (μ) , (v) and the geometrical parameters of the orifice, further permanent deformation of the continuum should continue to take place as long as:

$$P + \sigma_x \geq Y = Y_0 + K_0 \varepsilon^n \quad (13)$$

3.2 Pressure and Axial stress in the deformation zone

In the deformation zone the pressure gradient is slightly different, and can be expressed by equation (1). pressure values at different points at a distance (Δx) apart. It may be reasonable to assume that the plastic deformation takes place in a straight-line profile over small length (Δx) . Thus. It is assumed that deformation ceases at $x = \bar{x}$ where $\frac{dP}{dx} = 0$ where;

$$P_i = P_{i-1} + (p') \Delta x$$

substituting for (p') from equation (3)

$$P_i = P_{i-1} + \left(6\mu V \left(\frac{1}{h_i^2} - \frac{\bar{h}}{h_i^3} \right) \right) \Delta x \quad (14)$$

Where

$$h_i = h_{i-1} - (B - k_j) * \Delta x \quad (15)$$

$$D_i = D_{i-1} - 2k_j * \Delta x \quad (16)$$

$$V_i = V_{i-1} * \left(\frac{D_{i-1}}{D_i} \right) \quad (17)$$

Where, (k_j) is the linear deformation profile slope of the continuum. The equilibrium condition for a small element being deformed.

$$d\sigma_x = - \frac{2dD}{D} (Y + \tau_x \cot \alpha)$$

Where

$$D = D_1 - 2k_x, dD = -2k dx \text{ and } \cot \alpha = -1/k$$

the shear stress (τ_x) is given by

$$\tau_x = -\frac{h}{2} (p') - \frac{\mu V}{h}$$

Substituting for (p') from equation (3) and rearranging it becomes

$$\tau_x = \mu V \left(\frac{3\bar{h}}{h^2} - \frac{4}{h} \right)$$

hence the axial stress in the continuum can be expressed in finite difference form

$$d\sigma_x = \frac{4k dx}{D} \left\{ Y + \mu V \left(\frac{3\bar{h}}{h^2} - \frac{4}{h} \right) * \left(-1/k \right) \right\}$$

This is written in finite difference form as

$$\sigma_{xi} = \sigma_{xi-1} + \frac{4k_j \Delta x}{D_i} \left(Y_i - \frac{\tau_x}{k_j} \right) \quad (18)$$

Where

$$Y_i = Y_0 + K_0 \varepsilon^n$$

$$\text{Where } \varepsilon = 2 \ln \frac{D_1}{D_i} \text{ or } \varepsilon = \ln \left(\frac{D_1}{D_i} \right)^2$$

Therefore,

$$Y_i = Y_0 + K_0 \left\{ \ln \left(\frac{D_1}{D_i} \right)^2 \right\}^n \quad (19)$$

Deformation of the continuum continues as the plasto hydrodynamic. compatibility equation is satisfied.

$$P_i + \sigma_{xi} = Y_i$$

After completion of the deformation and the final shape of the diameter at a certain distance from the exit slot, pressure is calculated in this area, the same analysis before the distortion. by equation

$$P = \frac{6\mu V}{B} \left\{ \frac{1}{(h_1 - BX)} - \frac{\bar{h}}{2(h_1 - BX)^2} - \frac{1}{h_1} + \frac{\bar{h}}{2h_1^2} \right\} \quad (20)$$

The final value for (v) is used in equation (18) and (x) is any distance after the deformation has ceased.

The procedure for predicting the theoretical results involved the determination of (x_0), the distance from the entry of the orifice where plastic deformation commences and is found by iterative

computation of equations (2), (8) and Y_0 . From this point onward, the extent of the plastic deformation is known. Equations (12) and (16) for small increment ($x\Delta$) from (x_0).

The current slope of the deformation profile (k_j) is determined by iterative computation of equation (11), The deformation profile and percentage reduction in area are predicted for changes in drawing speed and orifice geometry the current diameter being then given by equation (14) The speed and gap change given by equation (15) and (13).

After the deformation has ceased the pressure can be determined from equation (18). The final value for (v) is used in equation (18) and (x) is any distance after the deformation has ceased

4. Results and discussion:

Fig (2) shows the hydrodynamic pressure profile generated in the orifice when pulling the wire at 0.3m/s. The effect Newtonian behavior can be observed with the steep increase in pressure approaching the maximum at 35mm along the orifice. A maximum pressure of 107 MPa

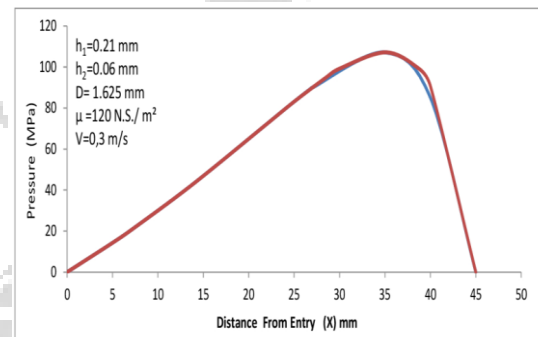


Fig (2) shows the results of pressure distribution at drawing speed (0,3m/s)

And fig (3) the maximum pressure was found to be (178.5MPa) at drawing speed of (0.5 m/s) and (357.14 MPa) at a drawing speed of (1 m/s). This figures suggests that the pressure increased from zero at the entry of the unit to maximum value when($x=35$ mm)

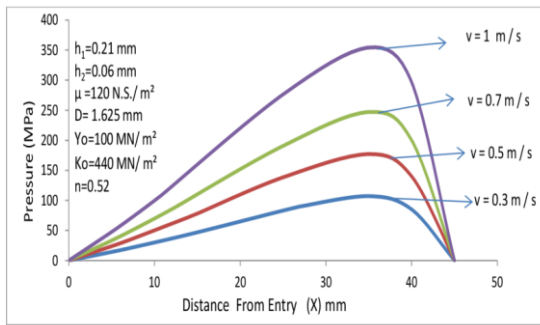


Fig (3) Effect of drawing speed on the pressure distribution within the unit

It can be seen from fig (3) and fig (4), that the plastic deformation starts and ends in the wire before the pressure reached its maximum value for various drawing speeds. This can be justified by knowing that as the wire is deformed a larger gap is then induced in the die unit. The newly added space in the die will be filled with the viscous fluid that needs a greater pressure to continue the polymer melt flow through the die.

The linear deformation profile of the wire is shown in fig (4) for various drawing speed (0.3-1 m/s) From the figure it can be seen that as the drawing speed increases the deformation in the wire increases

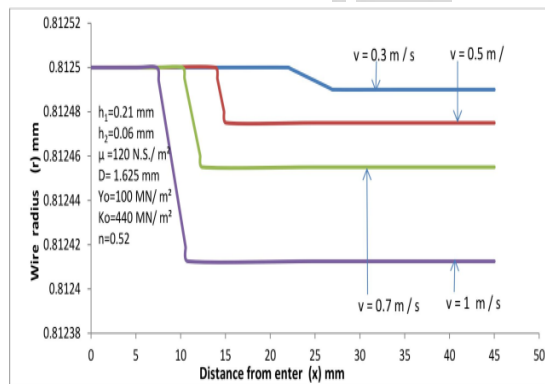


Fig (4) showing deformation profiles at different drawing speed

in speed of 0.5 m / s was to reduce the percentage in the area 0.0065% and which increases to 0.025% percent reduction in area at a drawing speed of 1m/s.

Fig (5) showing deformation profiles at different initial yield stress at drawing speed 0,5 m/s was to reduce the percentage in the area 0.0027% at μ equal 100 N.S./m and reduce the percentage in the area increases with increased viscosity .

Although there is a mismatch in the drawing speeds to attain a common percentage reduction in area, the difference is significantly more than the non Newtonian Solution [2].

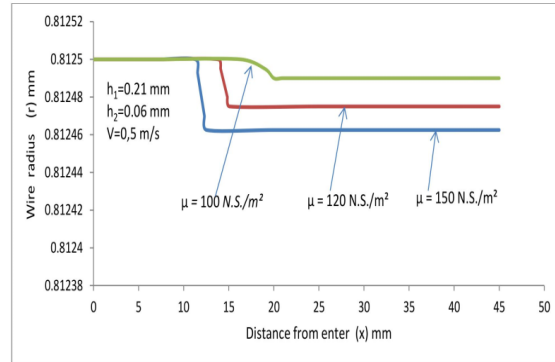


Fig (5) showing deformation profiles at different initial yield stress

Conclusions

In this study, a theoretical analysis based on the Newtonian fluid characteristics has been achieved. The main conclusions can be summarized as follows.

The plastic deformation starts and ends before the maximum pressure occurs.

The deformation profile of the wire is linear.

Deformation starts at a distance from the die entry as soon as Tresca hypothesis is valid.

As the drawing speed increases the deformation range decreases.

Nomenclature

- X is any distance (point) in the die.(mm)
- dx increment in length (mm)
- D diameter of wire.(mm)
- B slope for linear deformation
- P' pressure gradient in the unit. (N/m³)
- τ shear stress of the polymer melt in the orifice.
- μ viscosity of the fluid. (Ns/m²)
- V velocity of the wire (m/s)
- L length of the orifice(mm)
- U velocity of the fluid

P	hydrodynamic pressure. (N/m ²)
h	gap at any point in the orifice.(mm)
h₁	gap at the Inlet of the orifice(mm)
h₂	gap at the exit of the orifice(mm)
σ_x	is the axial pulling stress in the wire(N/m ²)
Y₀	initial yield stress (N/m ²)
Y	yield stress of the wire material (N/m ²)
K₀	strain hardening constant
	strain rate sensitivity constant

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