

## Effect of Spherical Capsules Diameter on the Performance of a Packed Bed for Cool Latent Heat Storage Systems

Zakareia A. Alarabi

Department of Petroleum Instrumentations, Higher Institute of Petroleum Technology, Awbari, LIBYA

### Abstract

This paper introduces a mathematical and numerical analyses of a packed bed containing spherical capsules filled with phase change material (PCM) for latent cool thermal energy storage (TES). The considered bed is cylindrical tank and containing spherical capsules arranged in a random form. According to the energy balance on the PCM and the heat transfer fluid (HTF), the governing equations and boundary conditions are derived at transient conditions and used a continuous phase model in this model; the spherical capsules are assumed to behave as a continuous medium. The governing equations of the HTF and PCM are discretized using the implicit finite difference approach and central difference approximation. The resulting algebraic equations were solved used MATLAB program. The numerical results are obtained to study the effects of spherical capsules diameter on the temperature distribution and energy stored in the bed in both charging and discharging modes.

**Keywords:** Packed bed, Phase change material, Spherical capsules, Thermal energy storage

### 1. Introduction

The storage of thermal energy is very important to many engineering applications. TES techniques are known from long time ago, but in latest years, more concern has been paid for the development of TES. Many studies have been done for PCM applications for latent TES, cold energy storage, enhancement of PCM thermal characteristics, and some applications of PCMs for thermal energy management are presented hereafter.

Ismail and Henriquez [1] presented a mathematical model for predicting the thermal performance of the cylindrical storage tank containing spherical capsules filled with water as PCM. The model was used to investigate the influence of the working fluid entry temperature, the flow rate of the working fluid and material of the spherical capsule of 77 mm diameter during the solidification process. The results shown that, the effect of varying the entry temperature of the working fluid Figure 2-3 from -3to-9°C has a very dominant and strong effect on the solidification time, while varying the temperature from -9to-15°C leads to

little variation of the time for complete solidification. In addition, Bilir and Ilken [2] investigated numerically the inward solidification problem of a PCM encapsulated in a cylindrical/spherical container with a third kind of boundary condition and ended up with a correlation which express the dimensionless total solidification time in terms of Stefan number, Biot number and superheat parameter. Moreover, Regin et al. [3] developed a theoretical model for analysis of the behavior of a packed bed consisting of spherical capsules filled with paraffin wax as PCM for latent TES system. A cylindrical storage tank has diameter of 1 m and height of 1.5 m, completely filled with PCM capsules. Were observed that the complete solidification time is longer than the melting time, and the capsule with a smaller radius has a significantly higher charging and discharging rates compared to those of larger radius. On the other hand S. Wu [4] presented a mathematical model of the cool TES system using packed bed containing spherical capsules filled with n-tetradecane to predict the thermal

behaviour of the this system. It was found lower porosity indicates higher cool storage capacity of packed bed, and hence longer time required for complete solidification and melting. The value of cool stored and released rate is much smaller at lower porosity. In addition, Cheralathan et al. [5] investigated numerically and experimentally a temperature profiles of HTF and PCM at any axial location and studied the influence of porosity during the charging process. Cool TES system comprised of a cylindrical storage tank filled with encapsulated PCMs in spherical container integrated with an aqueous solution of 30 wt. % ethylene glycol was used as HTF while distilled water with heterogeneous nucleation agents (super cooling release additive) was used as PCM. The results showed that, for lower porosity, the time averaged internal heat transfer coefficient and heat capacity of storage system are higher and longer time required for freezing the PCM. On the other hand ElGhnam et al. [6] investigated experimental study on the heat transfer during freezing and melting of water inside a spherical capsule of ice storage systems, Spherical capsules of different diameters and materials are tested. The results show that the energy recovery ratio is becoming better when using metallic capsules, increasing the capsule size and reducing the HTF volume flow rates. Furthermore Cho and Choi [7] investigated experimentally the thermal characteristics of spherical capsules using n-tetradecane, mixture of n-tetradecane and n-hexadecane and water as PCM, the study gives that the local phase-change temperatures of paraffin in a capsule were different during freezing and melting processes.

## Physical model .2

The physical system under study is shown in Figure 1. A cylindrical tank (called packed bed), the packed bed is insulated with polyurethane foams, and has an inner diameter of 1 m, a wall thickness of 0.05 m, and a height of 1.5 m. The PCM is encapsulated in spherical capsules packed randomly in the insulated cylindrical storage tank. Spherical capsules made of polyethylene,

has an inner diameter of 0.078 m and a thickness of 0.001 m. In this study, in the charging process, the HTF is charged from the bottom of the cool TES tank upwards, and flows over the spherical capsules in the cool TES tank. The HTF circulation is maintained by using a pump and exchanges energy with the PCM capsules and transfers the heat to the evaporator in a refrigeration system. In this process, PCM in the spherical capsules undergoes sensible cooling of liquid phase, solidification, sensible sub-cooling of solid phase. While in the discharging mode, the refrigeration system is not in operation. The HTF passes through the storage medium from the top of the cool TES tank downwards at temperature higher than the PCM fusion temperature, so heat transfer occurs. Energy released from the storage tank is controlled by flow control valves, and PCM in spherical capsules undergoes sensible heating of solid phase, melting, and sensible heating of liquid phase. In order to simplify the analysis, the following assumptions are considered:

- (1) The cool TES tank is well insulated.
- (2) The PCM and HTF thermo-physical properties are constant.
- (3) The heat transfer through the tank is one-dimensional along the height of the tank.
- (4) The velocity profile is assumed to be fully developed along the height of the tank.
- (5) The PCM has a constant solidifying and melting temperatures.
- (6) Radiant heat transfer between the capsules is negligible.

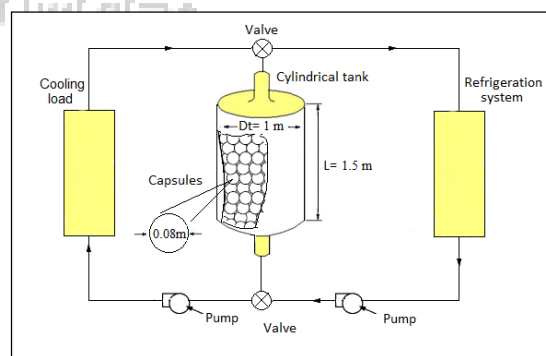


Figure 1 Layout and details of the packed bed cool TES system

### 3. Mathematical model

The conservation of mass, momentum, and energy can be written as following, respectively:

$$\frac{\partial \rho}{\partial \tau} = -(\nabla \cdot \rho \underline{v}) \quad 1$$

$$\frac{\partial(\rho \underline{v})}{\partial \tau} = -(\nabla \cdot \rho \underline{v} \underline{v}) - \nabla P + \mu \nabla^2 \underline{v} + \rho \underline{g} \quad 2$$

$$\frac{\partial(\rho c_p T)}{\partial \tau} = -(\nabla \cdot \rho c T \underline{v}) + k \nabla^2 T + q''' \quad 3$$

Where ( $\underline{g}$ ) is acceleration vector, ( $m/s^2$ ) and ( $q'''$ ) is the heat source or sink in ( $W/m^3$ ). The charging fluid flow is considered to be incompressible. Therefore, the mass conservation equation becomes:

$$\nabla \cdot \rho \underline{v} = 0 \quad 4$$

The fluid flow is assumed one-dimensional, thus continuity equation, (4) can be integrated to yield:

$$\rho_f (v_f)_y = constant \quad 5$$

in the y-direction a momentum equation reduces to the following form:

$$\left[ \rho_f (v_f)_y \right] \frac{d(v_f)_y}{dy} = - \frac{dP}{dy} \quad 6$$

the pressure drop across the packed bed [8] to geometrical parameters as follows:

$$\Delta P = \left[ \frac{150 \mu (1 - \varepsilon)^2}{d_o^2 \varepsilon^3} (v_f)_y + \frac{1.75 \rho_f (1 - \varepsilon)}{d_o \varepsilon^3} (v_f)_y^2 \right] L \quad 7$$

The energy balance equation on PCM and HTF can be written based on Shuangmao [4] model as:

$$\rho_f C_f \varepsilon \frac{\partial T_f}{\partial \tau} + \rho_f C_f \varepsilon v_f \frac{\partial T_f}{\partial y} = k_f \varepsilon \frac{\partial^2 T_f}{\partial y^2} + h_e a (\theta - T_f) \quad 8$$

The first term of the left hand side of Eq. (8) represents the rate change of internal energy of HTF, while the second term accounts for energy change due to the HTF flow. The two terms on the right hand represent the heat transfer by conduction and by convection between the HTF and the capsules, respectively. In the solidification stage, the PCM temperature  $\theta$  should be equal to freezing point ( $\theta_s$ ). The PCM temperature is also calculated by using energy balance on PCM and HTF.

Liquid phase stage (first stage):

$$\rho_l C_l (1 - \varepsilon) \frac{\partial \theta}{\partial \tau} = h_e a (T_f - \theta) \quad 9$$

Solidification stage (second stage):

$$\rho_s h_l (1 - \varepsilon) \frac{\partial \beta}{\partial \tau} = h_e a (\theta_s - T_f) \quad 10$$

Where  $L$ ,  $\beta$  and  $T_m$  denote latent heat of freezing, solid fraction and freezing temperature, respectively.

Solid phase stage (last stage):

$$\rho_s C_s (1 - \varepsilon) \frac{\partial \theta}{\partial \tau} = h_e a (T_f - \theta) \quad 11$$

Where ( $\varepsilon$ ) is the porosity of the bed, which was calculated using the correlation proposed by Beavers [9]:

$$\varepsilon = 0.4272 - 4.516 \times 10^{-3} \left( \frac{D}{d_o} \right) + 7.881 \times 10^{-5} \left( \frac{D}{d_o} \right)^2 \quad 12$$

And (a) is surface area per unit volume ( $m^{-1}$ )

$$a = \frac{6(1 - \varepsilon)}{d_o} \quad 13$$

The heat transfer coefficient between the spherical capsules and fluid was developed by Beek [10] for the case of capsules arranged in random form.

$$Nu = 3.22 Re^{1/3} Pr^{1/3} + 0.117 Re^{0.8} Pr^{0.4} \quad 14$$

The mean velocity, Reynolds number, Prandtl number of HTF can be obtained respectively from the relations,

$$v_f = \frac{\dot{V}}{\varepsilon A_t} \quad 15$$

$$Re = \frac{v_f \rho d_o}{\mu} \quad 16$$

$$Pr = \frac{c_f \mu}{k_f} \quad 17$$

The cool charge rate during charging process can be represented as,

$$Q_{ch} = \dot{V} \rho_f c_f (T_o - T_{in}) \quad 21$$

The cool charge capacity during charging process can be obtained as,

$$Q_{t,ch} = \int_0^{t_{ch}} Q_{ch} \cdot dt \quad 22$$

The cool discharge rate during charging process can be represented as,

$$Q_{disch} = \dot{V} \rho_f c_f (T_{in} - T_o) \quad 23$$

The cool discharge capacity during charging process can be obtained as,

$$Q_{t,disch} = \int_0^{t_{disch}} Q_{disch} \cdot dt \quad 24$$

#### 4. Boundary and Initial Conditions

The heat transfer coefficient is determined from

$$h = \frac{k_f Nu}{d} \quad 18$$

The solid-liquid interface can be calculated as follows,

$$r_p = [(1 - \beta)^{1/3}] \times r_i \quad 19$$

The effective coefficient of heat transfer can be determined from,

$$h_e = \frac{h}{\left(1 + \frac{hr_o(r_o - r_i)}{k_c r_i} + \frac{hr_o^2(r_i - r_p)}{k_s r_i r_p}\right)} \quad 20$$

The boundary condition can be expressed as follows:

$$\begin{aligned} T_f(y = 0) &= T_{in} && \text{for all } \tau \\ \text{And } \frac{\partial T_f}{\partial y}(y = L) &= 0 && \text{for all } \tau \end{aligned}$$

The second condition is the initial conditions for the first charging process.

$$\text{At } T_f(\tau = 0) = T_{in} \quad \text{for } y=0$$

$$T_f(\tau = 0) = T_o \quad \text{for } y \neq 0$$

$$\theta(\tau = 0) = T_o \quad \text{for } y \neq 0$$

On the other hand, when the bed is discharged, the boundary condition becomes:

At  $T_f(y = 0) = T_o$  for all  $\tau$

$$\omega_i^n = \frac{h_{ei}^n a}{\rho L(1 - \varepsilon)}$$

And  $\frac{\partial T_f}{\partial y}(y = L) = 0$  for all  $\tau$

And the initial condition, in the discharging process

At  $T_f(\tau = 0) = T_o$  for  $y=0$

$T_f(\tau = 0) = T_{in}$  for  $y \neq 0$

$\theta(\tau = 0) = T_{in}$  for  $y \neq 0$

$$(A - B)T_{i+1}^{n+1} + (1 + 2B + C_i^n)T_i^{n+1} + (-A - B)T_{i-1}^{n+1} = T_i^n + C_i^n \theta_i^n \quad 19$$

Where

$$A = \frac{v_f \Delta \tau}{2 \Delta x} \quad 28$$

$$B = \frac{k_f \Delta \tau}{\rho_f C_f \Delta x^2} \quad 20$$

$$C_i^n = \frac{\omega_{fi}^n \Delta \tau}{1 + \omega_i^n \Delta \tau} \quad \text{One phase of PCM} \quad 30$$

$$C_i^n = \omega_{fi}^n \Delta \tau \quad \text{Two phases of PCM} \quad 31$$

Where

$$\omega_i^n = \frac{h_{ei}^n a}{\rho_l C_l (1 - \varepsilon)} \quad \text{Liquid phase} \quad \omega_{fi}^n = \frac{h_{ei}^n a}{\rho_f C_f \varepsilon} \quad 32$$

$$\omega_i^n = \frac{h_{ei}^n a}{\rho_s C_s (1 - \varepsilon)} \quad \text{Solid phase}$$

Table 1 shows the physical and thermal properties of PCMs and HTF.

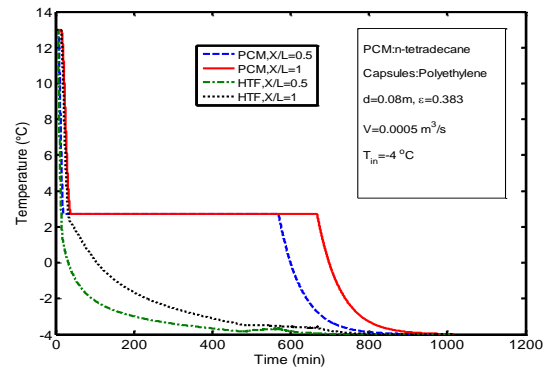
With Phase change stage (10):

$$\beta_i^{n+1} = \beta_i^n + \omega_i^n \Delta \tau (\theta_s - T_i^{n+1}) \quad 26$$

Where

**Table -1** Physical and Thermal properties of PCMs and HTF

Properties	Phase change materials		(HTF) Ethylene glycol 40% Vol
	n-tetradecane		
	Solid	Liquid	
Solidifying temperature, °C	2.73	-	-
Melting temperature, °C	7.79	-	-
Solidifying latent heat, kJ/kg	213.83	-	-
Specific heat, kJ/kg.K	2.00	2.55	3.45
Thermal conductivity, W/m.K	0.273	0.211	0.44
Density, kg/m <sup>3</sup>	803	765	1070
Dynamic Viscosity, kg/m.s	-	-	0.00906



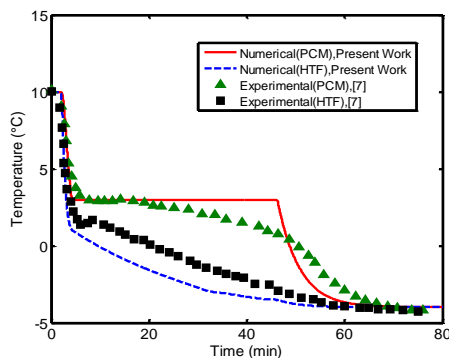
**Figure -3** Temperatures distribution of PCM and HTF at middle and outlet of packed bed with time during charging process

## 6. Validation of the present work

Figure 2 presents the variations with time for the numerical and measured temperatures of PCM, and HTF at the outlet locations. As can be seen in Figure 2, there is good agreement between the present numerical results and the experimental data in [7].

## 7. Charging process of packed bed

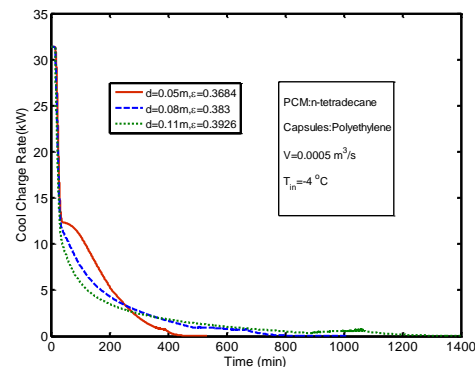
The numerical simulations are conducted at various diameters of capsules (0.05-0.11m), figure 3 shows the variation of the temperatures of PCM and HTF with time at middle and outlet positions of the packed bed. It is clearly seen that the PCM undergoes three stages during the charging process, namely, liquid sensible cooling stage, phase change (freezing) stage, and solid sub-cooling stage.



**Figure -2** Comparison between the present model and the experimental study [7]

The time for complete freezing at the middle location is about 569 min (9:29 hr) and at the outlet is about 667 min (11:7 hr). This is due to the fact that heat transfer rate decreases as the temperature difference between PCM and HTF decreases along the flow direction of HTF. So, it is clearly seen that the closer to the outlet position, the longer time for complete solidification and charging process. Also note that the total charging time process of the system is 1015 min (16:55 hr).

Figure 4 represents variation of cool charge rate with time at different diameter of capsules, a reduction of the capsule diameter results in an increase in the number of capsules and in the heat transfer area, and hence the heat transfer rate between the fluid and capsules increases.



**Figure -4** Variation of cool stored rate with time at different diameter of capsules

Figure 5 shows the effect of capsules diameter on the time for complete charging at different capsules diameter. The smaller capsules diameter has less time for complete charging. Capsules with 0.08 m diameters have time for complete charging of 1015 min (16:55 hr). When capsules diameter decreased to 0.05 m the time for complete charging is decreased by 47%, to 533 min (8:53 hr), and the increased of the capsules diameter to 0.11 m due the increased in time for complete charging by 67.9%, to 1705 min (28:25 hr).

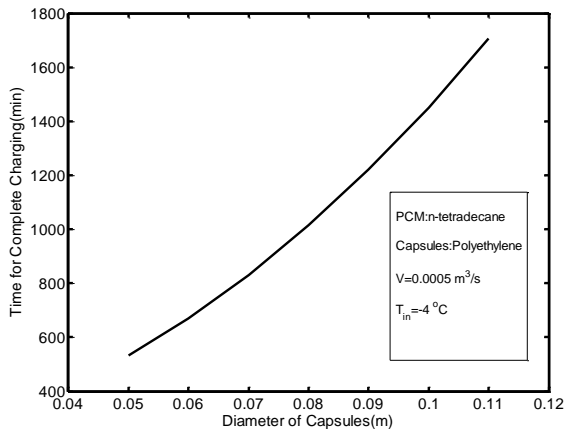


Figure -5 Effect of diameter of capsules on time for complete charging

Figure 6 illustrates variation of cool charge capacity with time at different diameters of capsules. It is shown that if the diameter of capsules increases, the porosity is increased for the same bed diameter. Therefore, the mass of PCM is decreased. Thus, the capacity charged in the bed is decreased by increasing the diameter of capsules.

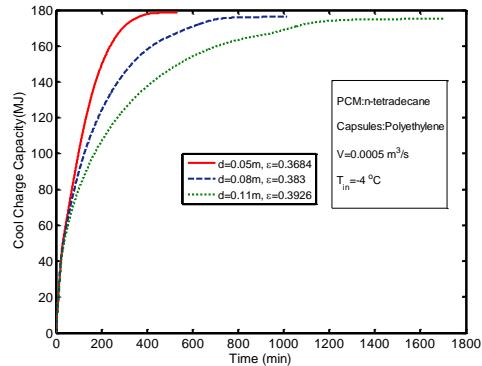


Figure -6 Variation of cool charge capacity with time at different diameter of capsules

The case of 0.08 m diameter has a capacity of 176 MJ; at the small case of 0.05 m diameter which has a largest capacity 179 MJ as changed by 1.7%, and at the case of 0.11 m diameter which has 175 MJ of capacity as changed by 0.56%.

Figure 7 shows the spatial pressure drop across the packed bed at different diameter of capsules, using the Ergun equation (7).

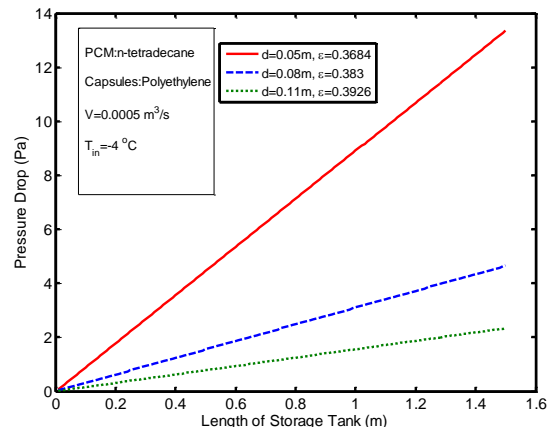


Figure -7 Variation of pressure drop along the bed length at different diameter of capsules

The results obtained that, the rate of pressure drop across the packed bed in the case of 0.08 m capsules diameter with 0.3684 porosity was 3.099(Pa/m), it can be noticed that increasing the diameter of capsules to 0.11 m with 0.3926 porosity results in a decrease in the rate of pressure drop across the packed bed to 1.558(Pa/m) with percentage decrease of 49.72%, this is because increasing the diameter of capsules leads to increasing the porosity as shown in equation (12), this leads to a decrease in the pressure drop across the packed bed. While the rate of pressure drop increased with decreased the diameter of capsules to 0.05 m and 0.3926 porosity, which was 8.907(Pa/m), the increase was approximately three times.

### 8. Discharging process of packed bed

Figure 8 displays the variation of the temperatures of PCM and HTF with time at middle and outlet positions of the packed bed during discharging process. It is clearly seen that the PCM undergoes three stages during the discharging process, namely, solid sensible heating stage, phase change (melting) stage, and liquid sensible heating stage. The time for complete melting at the middle location is about 895 min (14:55 hr) and at the outlet is about 1020 min (17:00 hr). This is due to the fact that heat transfer rate decreases as the temperature difference between PCM and HTF decreases along the flow direction of HTF. So, it is clearly seen that the closer to the outlet position, the longer time for complete melting and discharging process. Also note that the total discharging time process of the system is 1466 min (24:26 hr).

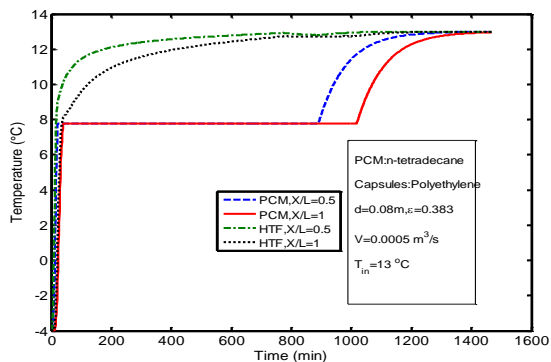


Figure -8 Temperatures distribution of PCM and HTF with time during discharging process

Figure 9 represents variation of cool discharge rate with time at different diameter of capsules. A reduction of the capsule diameter induces an increase in the number of capsules and in the heat transfer area, and hence the heat transfer rate between the fluid and capsules increases.

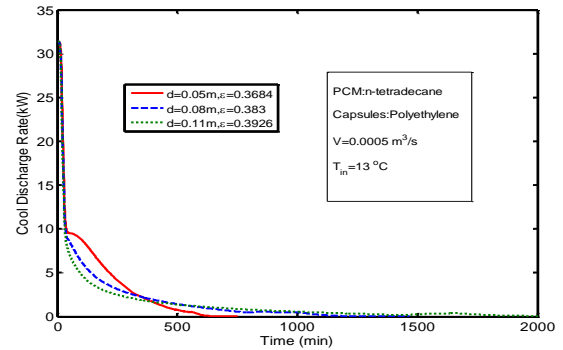


Figure -9 Variation of cool discharge rate with time at different diameter of capsules

Figure 10 displayed the effect of capsules diameter on time for complete discharging at different capsules diameter. The smaller capsules diameter has less time for complete discharging. Capsules with 0.08 m have time for complete discharging of 1466 min (24:26 hr), with decreasing the capsules diameter to 0.05 m which due to decreasing time for complete discharging to 747 min (12:27 hr) by percentage of decreasing equal to 49.04%, whereas when increase the capsules diameter to 0.11 m that due to increasing time for complete discharging to 2499 min (41:39 hr), the time for complete discharging is increased by 70.4%.

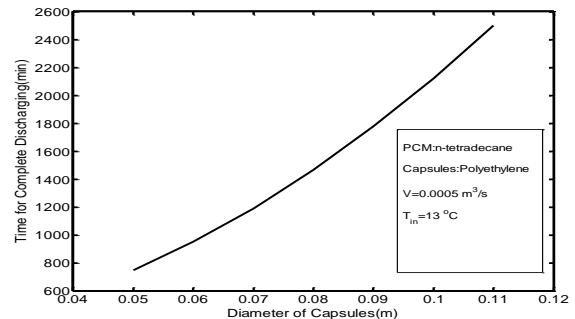


Figure -10 Effect of diameter of capsules on time for complete discharging



Figure 11 illustrates variation of cool discharge capacity with time at different diameter of capsules. It is shown that if the diameter of capsules increases, the porosity is increased for the same bed diameter. Therefore, the mass of PCM is decreased. Thus, the capacity discharged in the bed is decreased by increasing the diameter of capsules. The case of 0.08 m diameter has a capacity of 174 MJ; at the small case of 0.05 m diameter which has a largest capacity 177 MJ as changed by 1.7%, and at the case of 0.11 m diameter which has 172 MJ of capacity as changed by 1.14%.

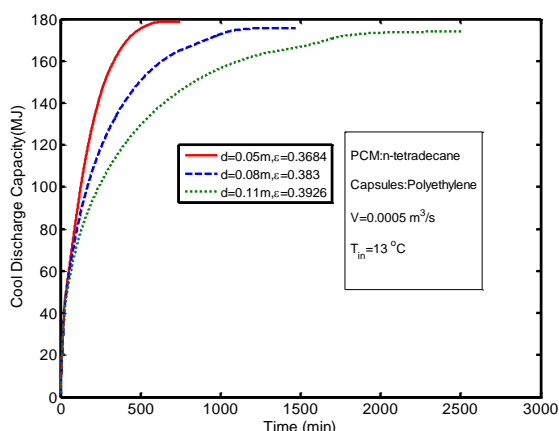


Figure -11 Variation of cool discharge capacity with time at different diameter of capsules

## 9. Conclusion

Increase the spherical capsules diameter for the same bed diameter results in an increase in the time for complete charging and discharging. Moreover, it results in a decrease in the average charge and discharge rate. Also, it results in a decrease in a capacity and in pressure drop across the packed bed. The diameter of capsules has large effect on the time for complete charging and discharging, as a diameter of capsules varied from 0.05-0.11m, the time for complete charging changes from 533-1705 min, while the time for complete discharging changes from 747-2642 min, and also the pressure drop is varied from 8.907 to 1.558(Pa/m).

## List o Symbols

$A_t$	Cross-section area of the cylindrical tank ( $m^2$ )
$a$	Surface area of spherical capsules per volume ( $1/m$ )
$c$	Specific heat ( $J/kg.K$ )
$D$	Inner diameter of packed bed (m)
$d$	Diameter of capsule (m)
$h$	Coefficient of convective heat transfer ( $W/m^2.K$ )
$h_e$	Effective coefficient of heat transfer ( $W/m^2.K$ )
$h_l$	Latent heat ( $J/kg$ )
$k$	Thermal conductivity ( $W/m.K$ )
$L$	Height of Packed bed (m)
$Nu$	Nusselt number of HTF
$P$	Pressure ( $N/m^2$ )
$Pr$	Prandtl number of HTF
$Q$	Cool charge and discharge rate ( $W$ )
$Q_t$	Cool charge and discharge capacity ( $J$ )
$Re$	Reynolds number of HTF
$r$	Radius of capsule (m)
$T$	Temperature of HTF ( $^{\circ}C$ )
$\dot{V}$	Volume flow rate of HTF ( $m^3/s$ )
$\beta$	Solid and melt fraction
$\Delta$	Increment of variable
$\varepsilon$	Porosity of bed
$\theta$	Temperature of PCM ( $^{\circ}C$ )
$\mu$	Viscosity ( $kg/m.s$ )
$v$	Mean velocity of HTF ( $m/s$ )
$\rho$	Density ( $kg/m^3$ )
$\tau$	Time (s)

## Subscripts

$C$	Cover of capsules
$ch$	Charge
$disch$	Discharge
$f$	Heat transfer fluid
$i$	Inner radius
$in$	Inlet position
$l$	Liquid phase
$o$	Outer radius
$s$	Solid phase
$t$	Tank

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